UNIVERSITÄT OSNABRÜCK

A Thorough Formalization of Conceptual Spaces

Lucas Bechberger and Kai-Uwe Kühnberger

The Different Layers of Representation

The Different Layers of Representation

Symbolic Layer $\forall x: \text{apple}(x) \Rightarrow \text{red}(x)$

Formal Logics

The Different Layers of Representation

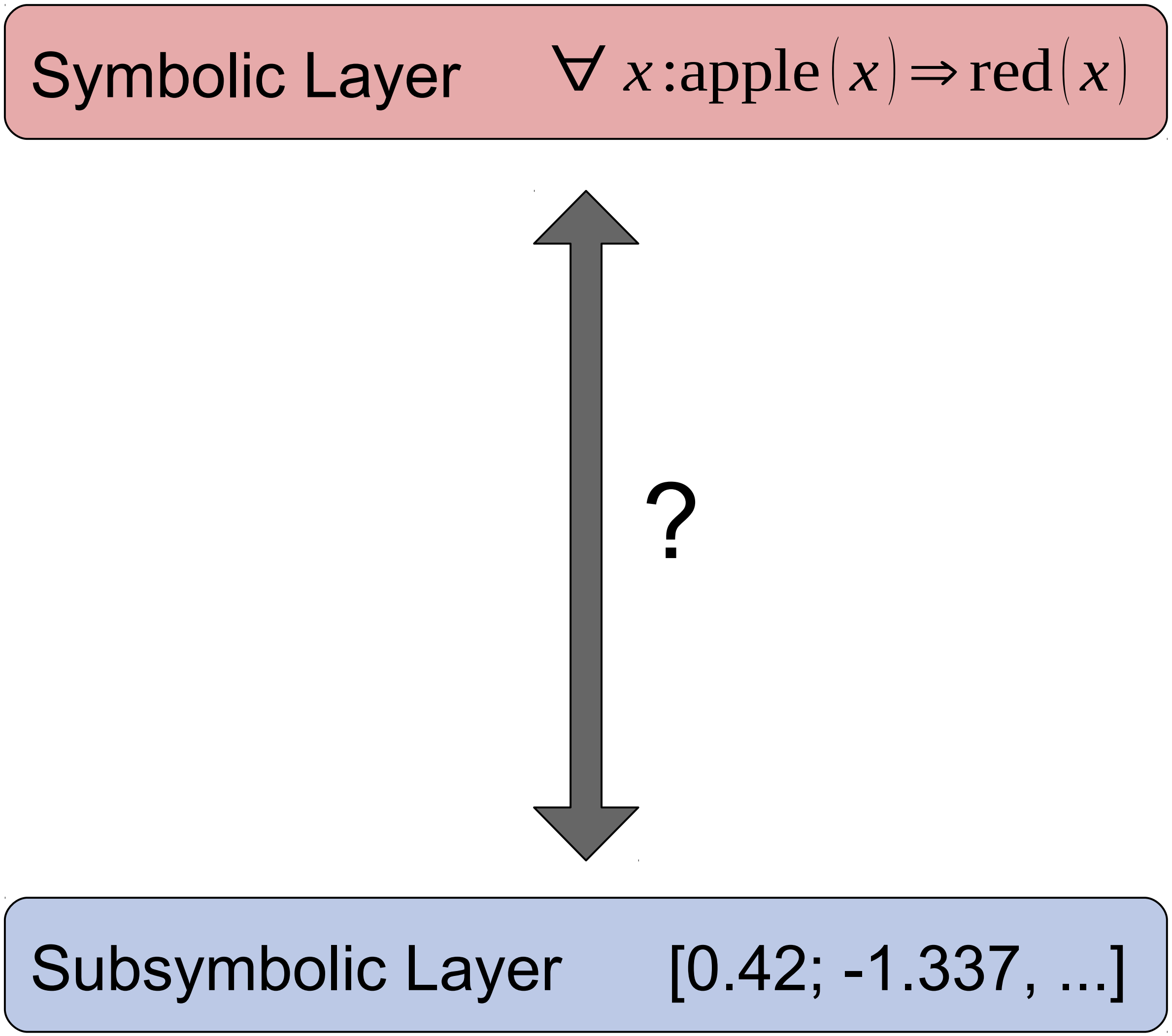
Symbolic Layer $\forall x : \text{apple}(x) \Rightarrow \text{red}(x)$

Formal Logics

Subsymbolic Layer [0.42; -1.337, ...]

Sensor / Feature Values

The Different Layers of Representation



Formal Logics

Sensor / Feature Values

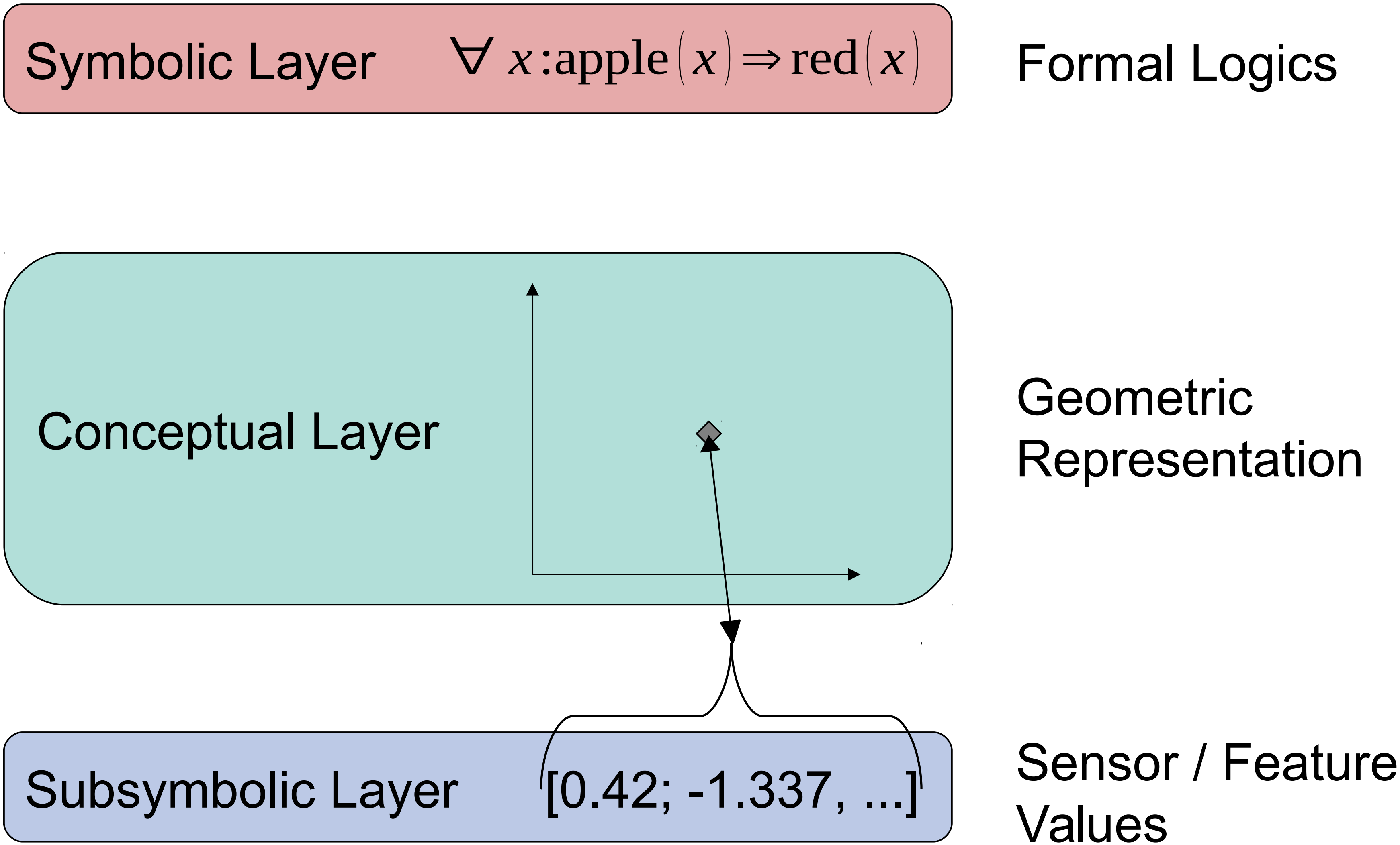
The Different Layers of Representation

Symbolic Layer $\forall x : \text{apple}(x) \Rightarrow \text{red}(x)$ — Formal Logics

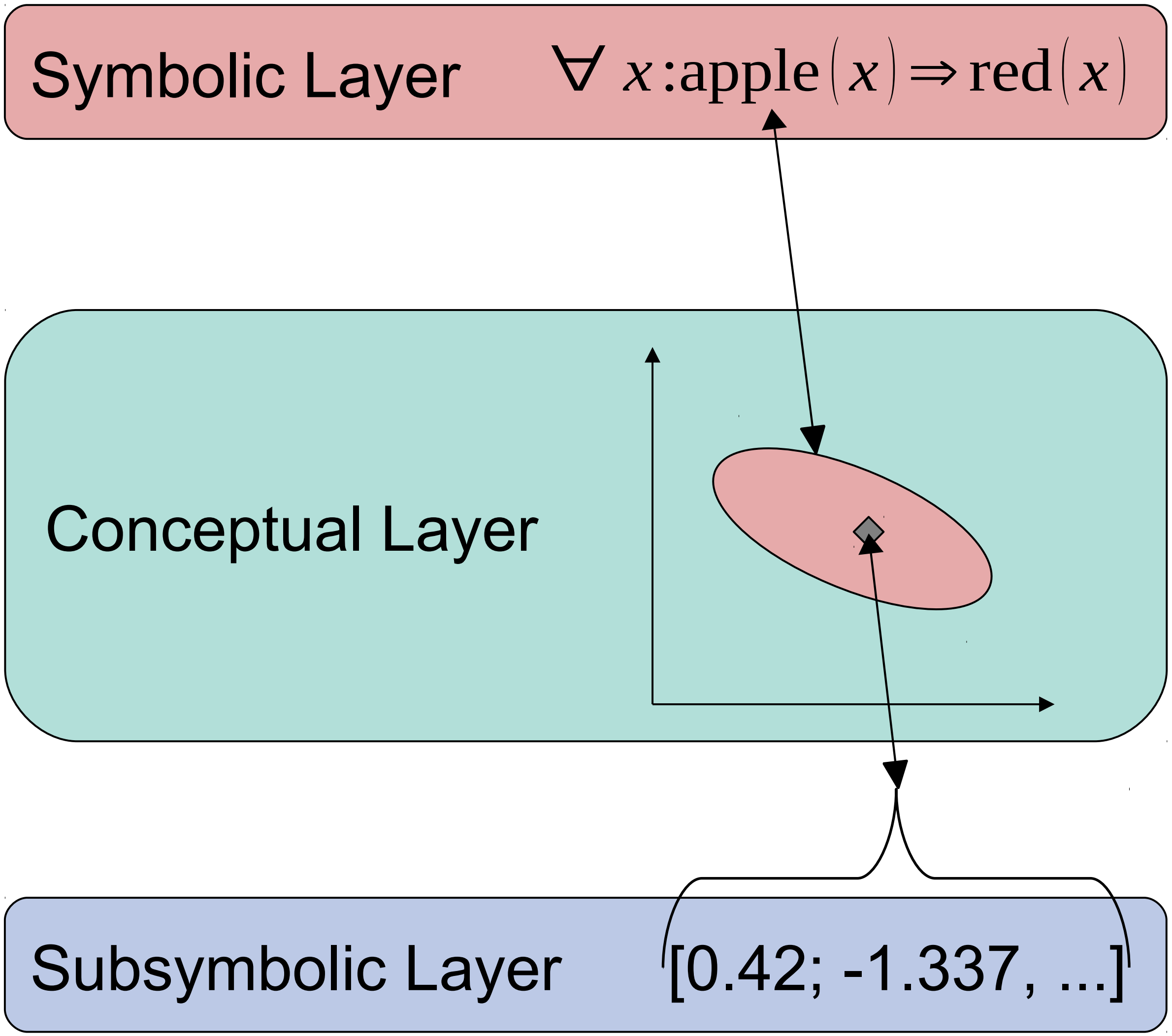
Conceptual Layer — Geometric Representation

Subsymbolic Layer [0.42; -1.337, ...] — Sensor / Feature Values

The Different Layers of Representation



The Different Layers of Representation



Formal Logics

Geometric Representation

Sensor / Feature Values

Conceptual Spaces [Gärdenfors2000]

[Gärdenfors2000] Gärdenfors, P. Conceptual Spaces: The Geometry of Thought. *MIT press*, 2000

Conceptual Spaces [Gärdenfors2000]

- Quality dimensions
  - Interpretable ways of judging the similarity of two instances
  - E.g., temperature, weight, brightness, pitch

[Gärdenfors2000] Gärdenfors, P. Conceptual Spaces: The Geometry of Thought. *MIT press*, 2000

Conceptual Spaces [Gärdenfors2000]

- Quality dimensions
  - Interpretable ways of judging the similarity of two instances
  - E.g., temperature, weight, brightness, pitch
- Domain
  - Set of dimensions that inherently belong together
  - Color: hue, saturation, and brightness

[Gärdenfors2000] Gärdenfors, P. Conceptual Spaces: The Geometry of Thought. *MIT press*, 2000

Conceptual Spaces [Gärdenfors2000]

- Quality dimensions
  - Interpretable ways of judging the similarity of two instances
  - E.g., temperature, weight, brightness, pitch
- Domain
  - Set of dimensions that inherently belong together
  - Color: hue, saturation, and brightness
- Distance in this space is inversely related to similarity
  - Within a domain: Euclidean distance
  - Between domains: Manhattan distance

[Gärdenfors2000] Gärdenfors, P. Conceptual Spaces: The Geometry of Thought. *MIT press*, 2000

Conceptual Spaces [Gärdenfors2000]

- Quality dimensions
  - Interpretable ways of judging the similarity of two instances
  - E.g., temperature, weight, brightness, pitch
- Domain
  - Set of dimensions that inherently belong together
  - Color: hue, saturation, and brightness
- Distance in this space is inversely related to similarity
  - Within a domain: Euclidean distance
  - Between domains: Manhattan distance
- Concepts
  - Region + correlation information + salience weights

[Gärdenfors2000] Gärdenfors, P. Conceptual Spaces: The Geometry of Thought. *MIT press*, 2000

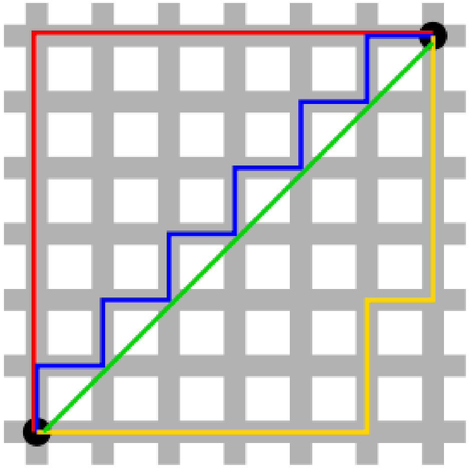
Betweenness

Betweenness

- $B(x,y,z) :\leftrightarrow d(x,y) + d(y,z) = d(x,z)$

Betweenness

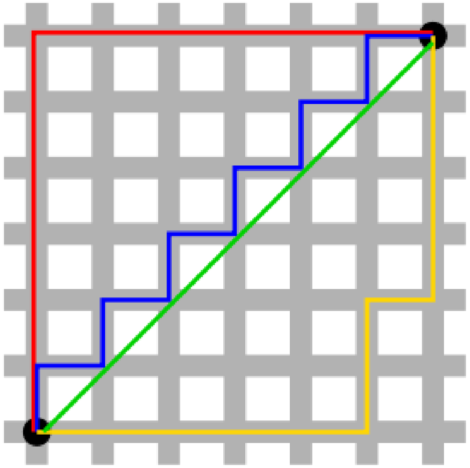
- $B(x,y,z) :\leftrightarrow d(x,y) + d(y,z) = d(x,z)$



https://en.wikipedia.org/wiki/Taxicab_geometry#/media/File:Manhattan_distance.svg

Betweenness

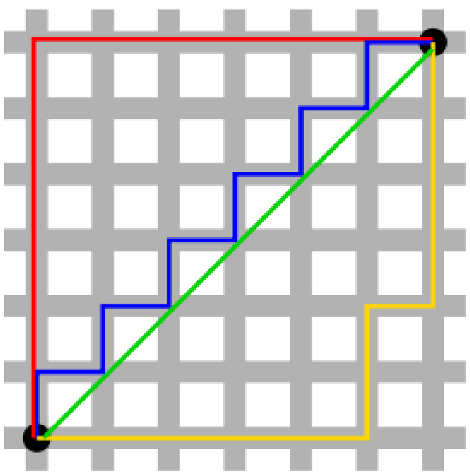
- $B(x,y,z) :\leftrightarrow d(x,y) + d(y,z) = d(x,z)$



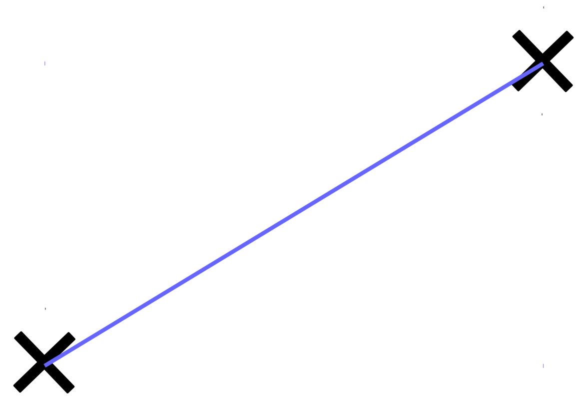
https://en.wikipedia.org/wiki/Taxicab_geometry#/media/File:Manhattan_distance.svg

Betweenness

- B(x,y,z) :↔ d(x,y) + d(y,z) = d(x,z)

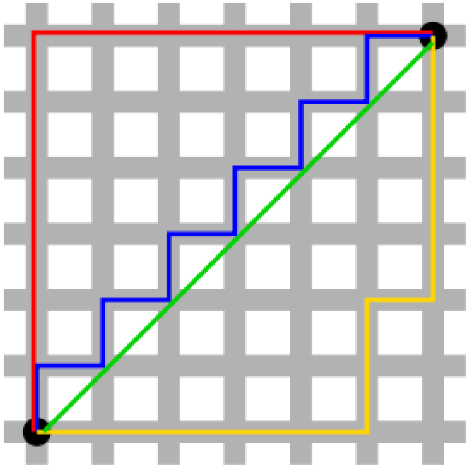


https://en.wikipedia.org/wiki/Taxicab_geometry#/media/File:Manhattan_distance.svg

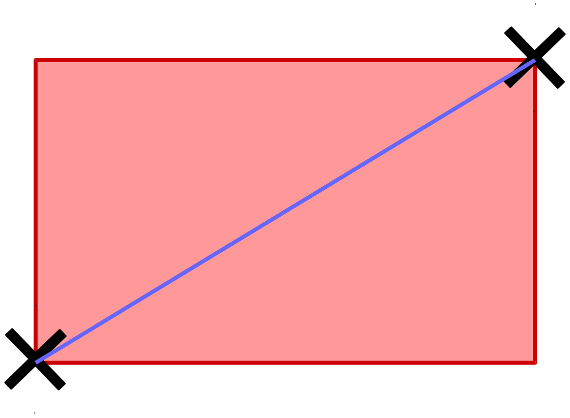


Betweenness

- $B(x,y,z) :\leftrightarrow d(x,y) + d(y,z) = d(x,z)$

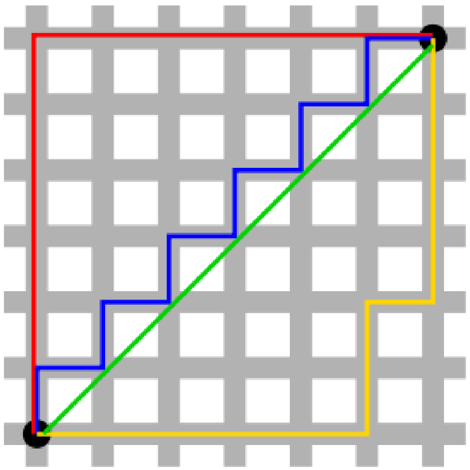


https://en.wikipedia.org/wiki/Taxicab_geometry#/
media/File:Manhattan_distance.svg

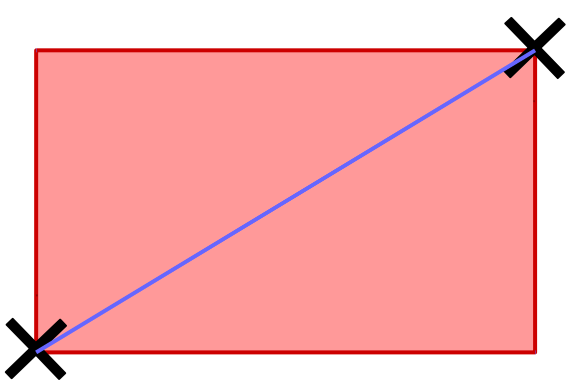


Betweenness

- B(x,y,z) :↔ d(x,y) + d(y,z) = d(x,z)
- Convex region C: $\forall\, x,z \in C : \forall\, y : B(x,y,z) \Rightarrow y \in C$

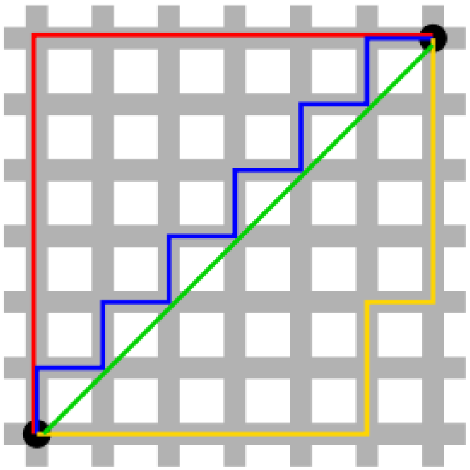


https://en.wikipedia.org/wiki/Taxicab_geometry#/media/File:Manhattan_distance.svg

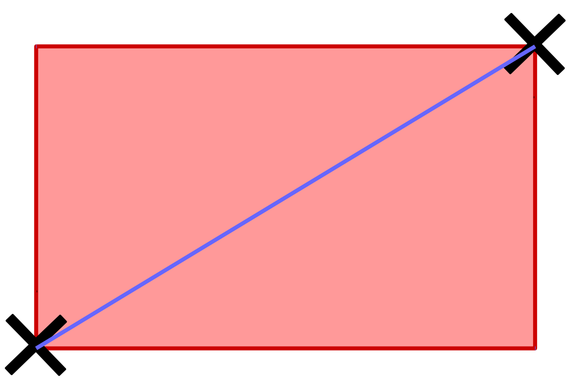


Betweenness

- B(x,y,z) :↔ d(x,y) + d(y,z) = d(x,z)
- Convex region C: $\forall\, x,z \in C : \forall\, y : B(x,y,z) \Rightarrow y \in C$
- Star-shaped region S w.r.t. p: $\forall\, z \in S : \forall\, y : B(p,y,z) \Rightarrow y \in S$

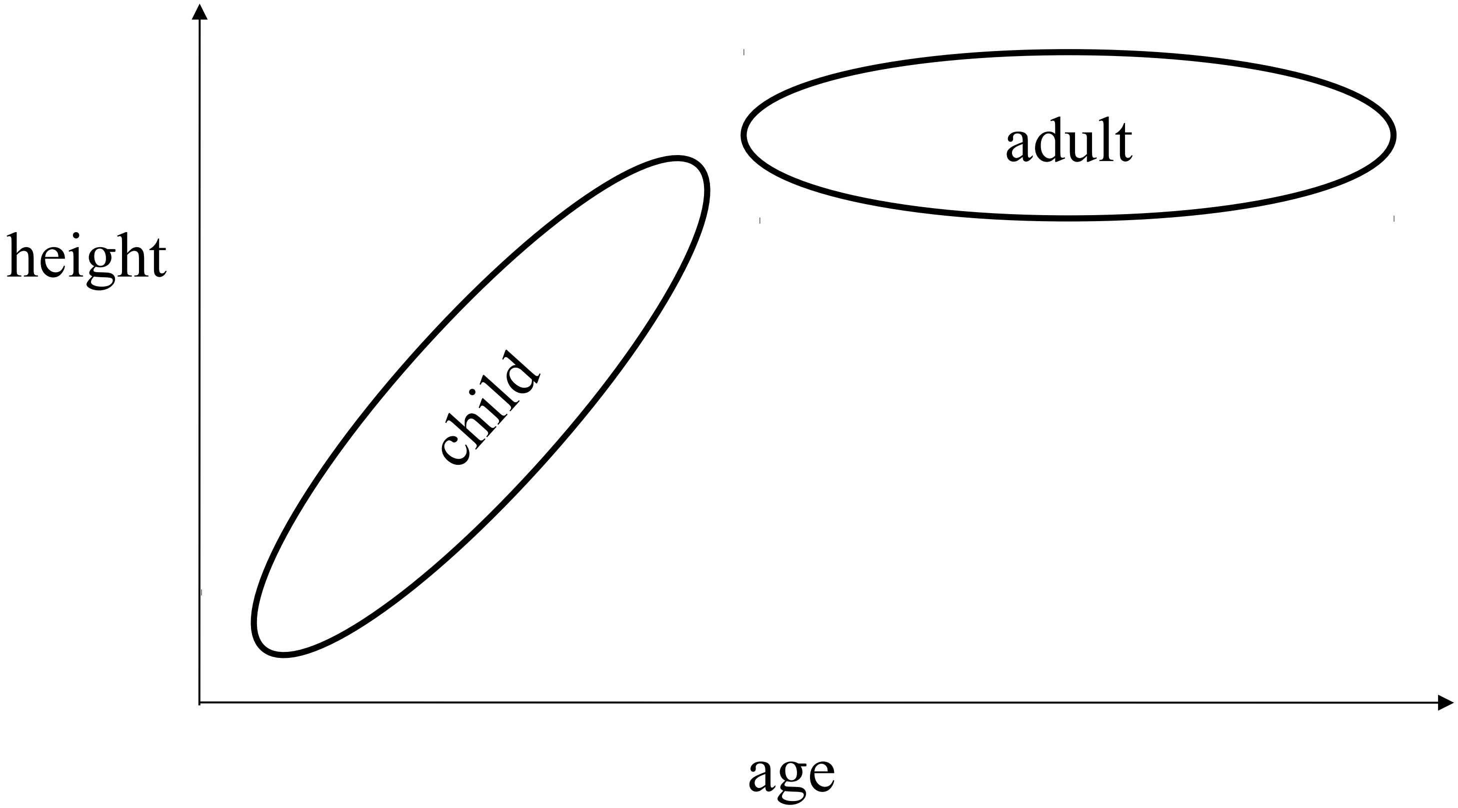


https://en.wikipedia.org/wiki/Taxicab_geometry#/media/File:Manhattan_distance.svg

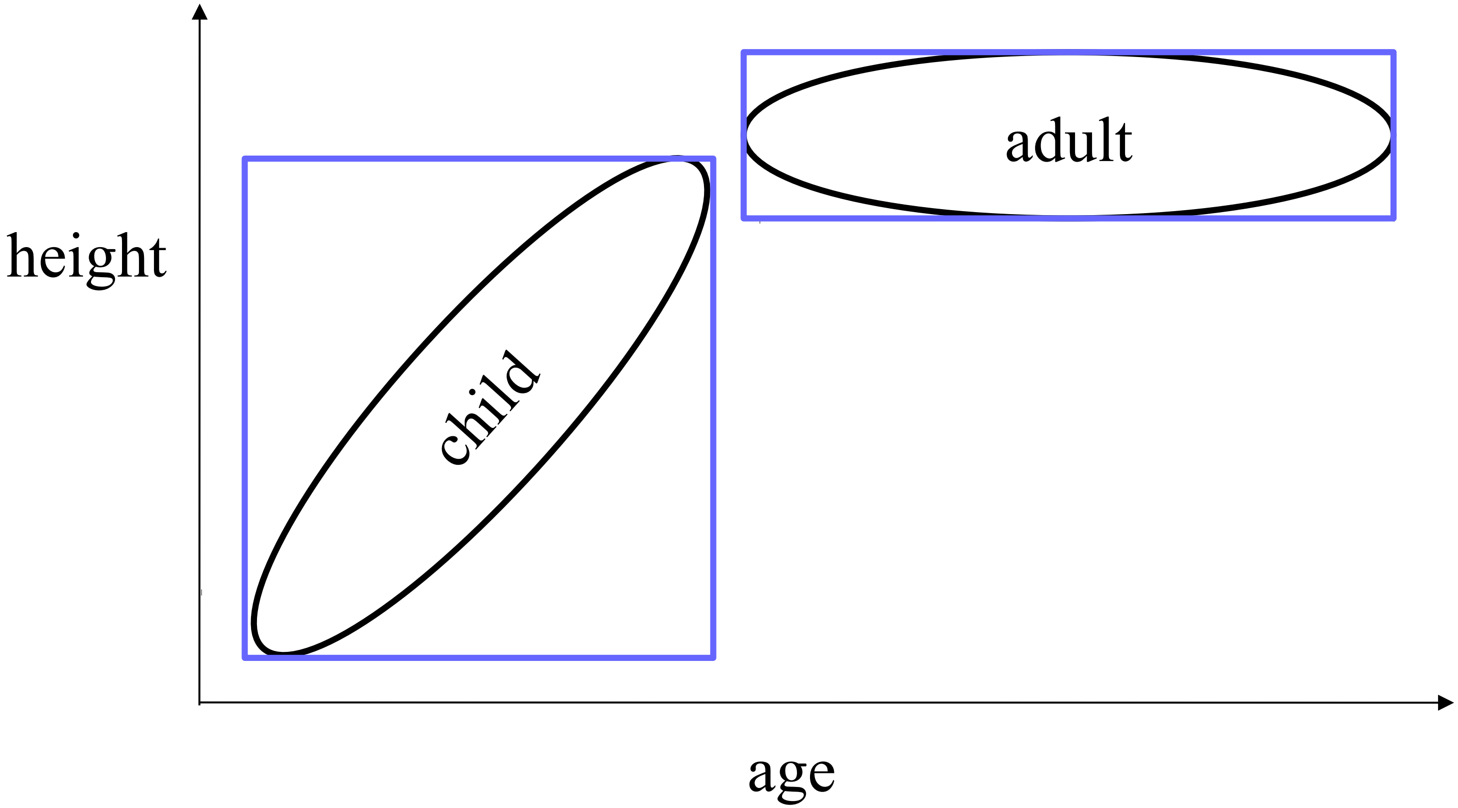


Convexity and Manhattan distance

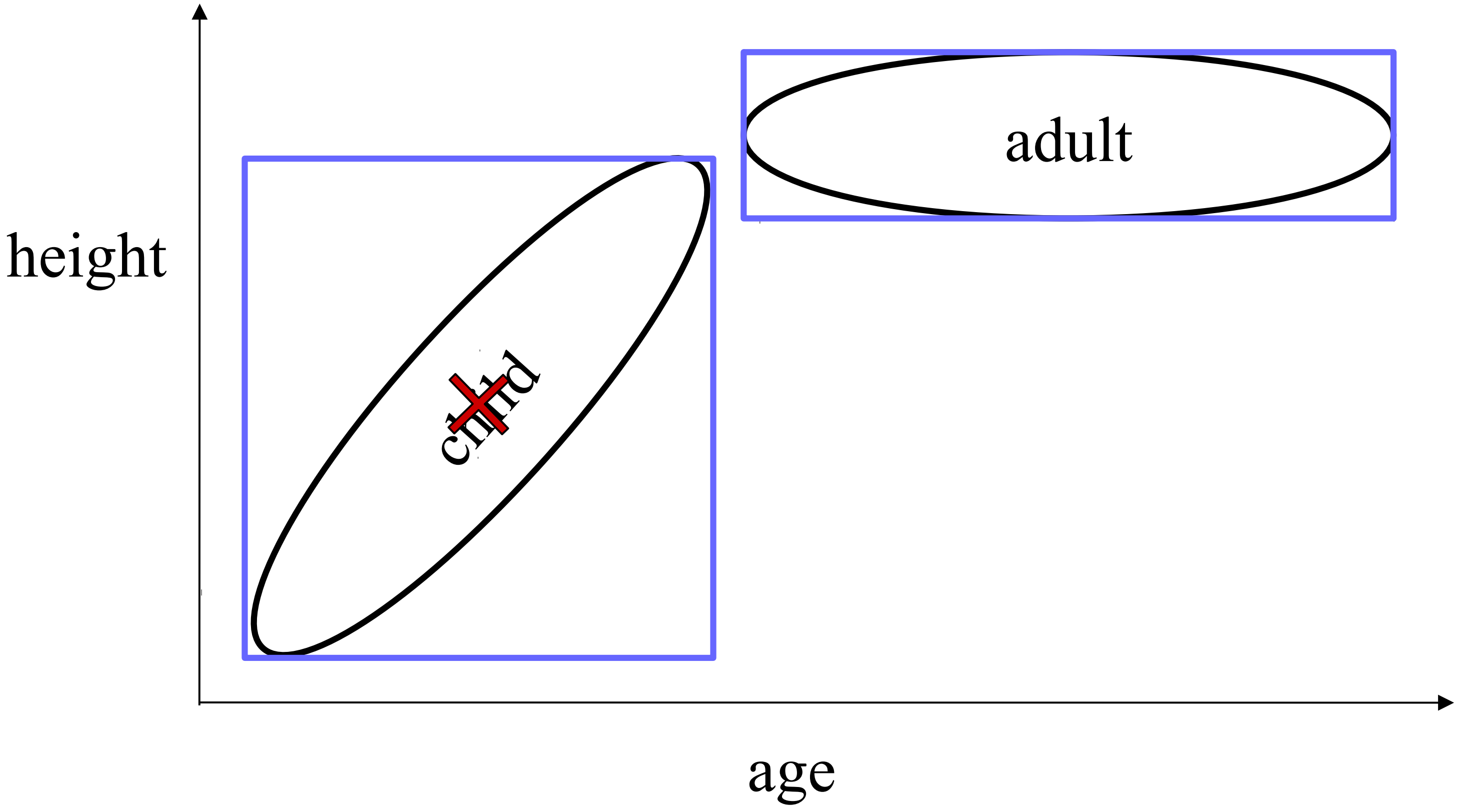
Convexity and Manhattan distance



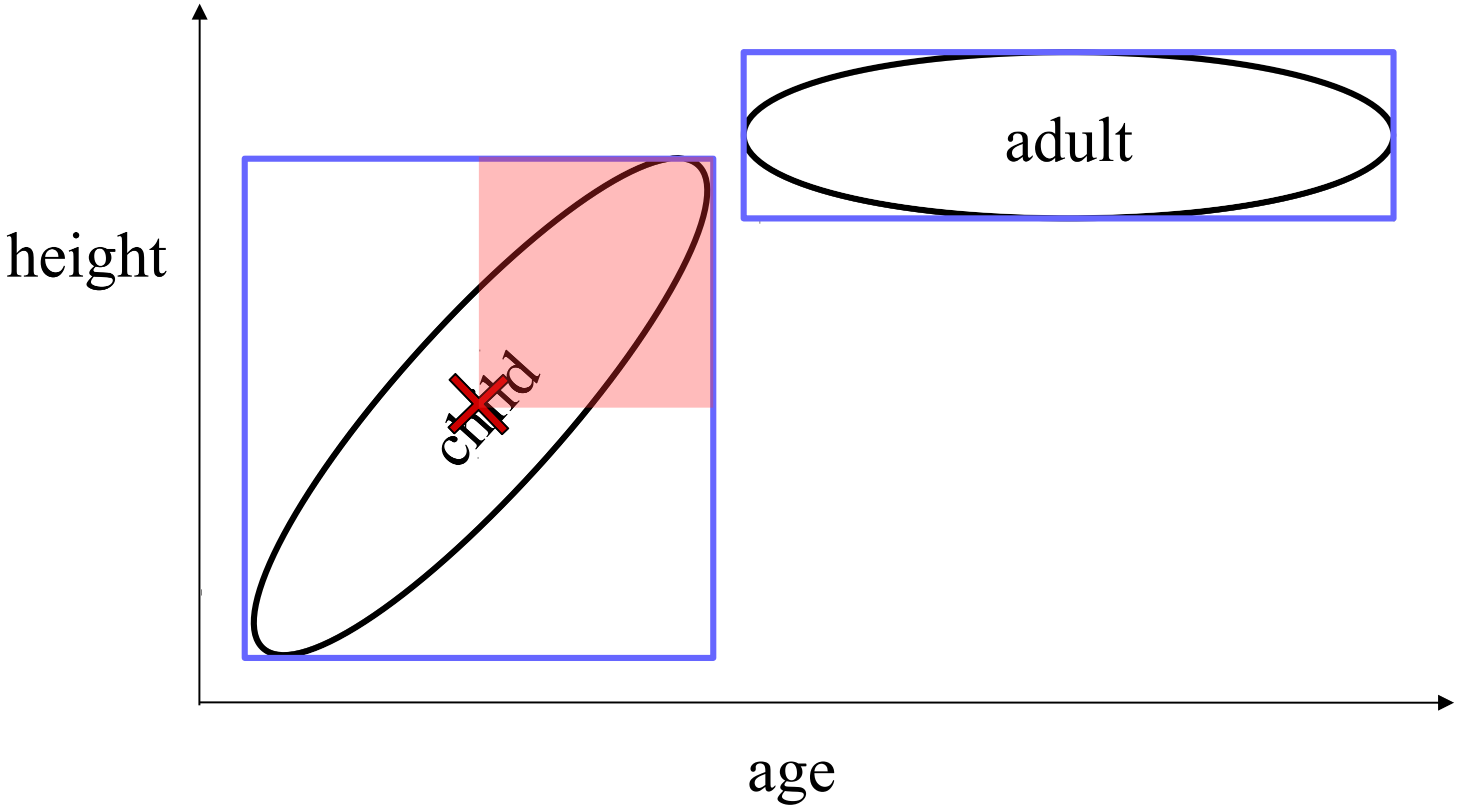
Convexity and Manhattan distance



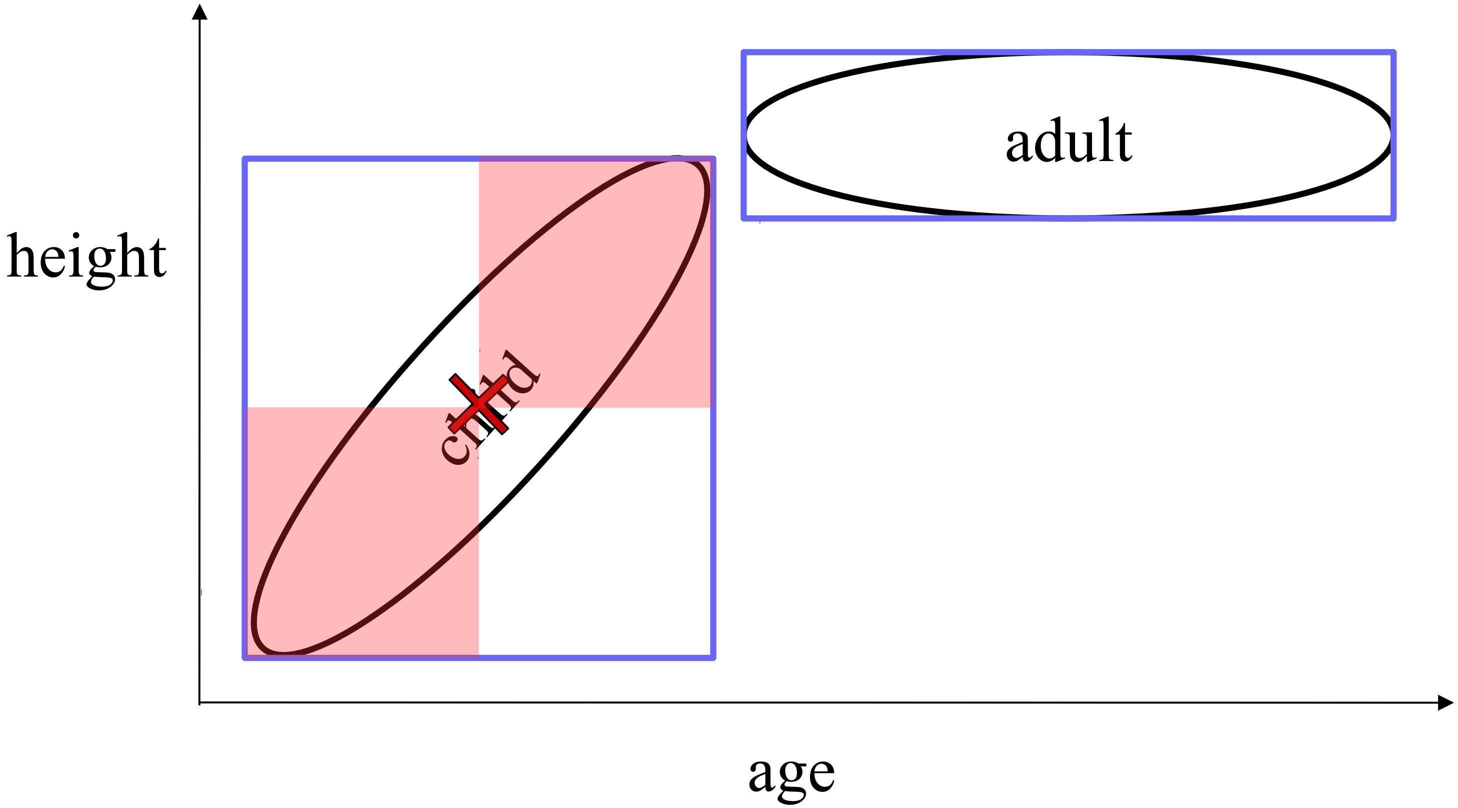
Convexity and Manhattan distance



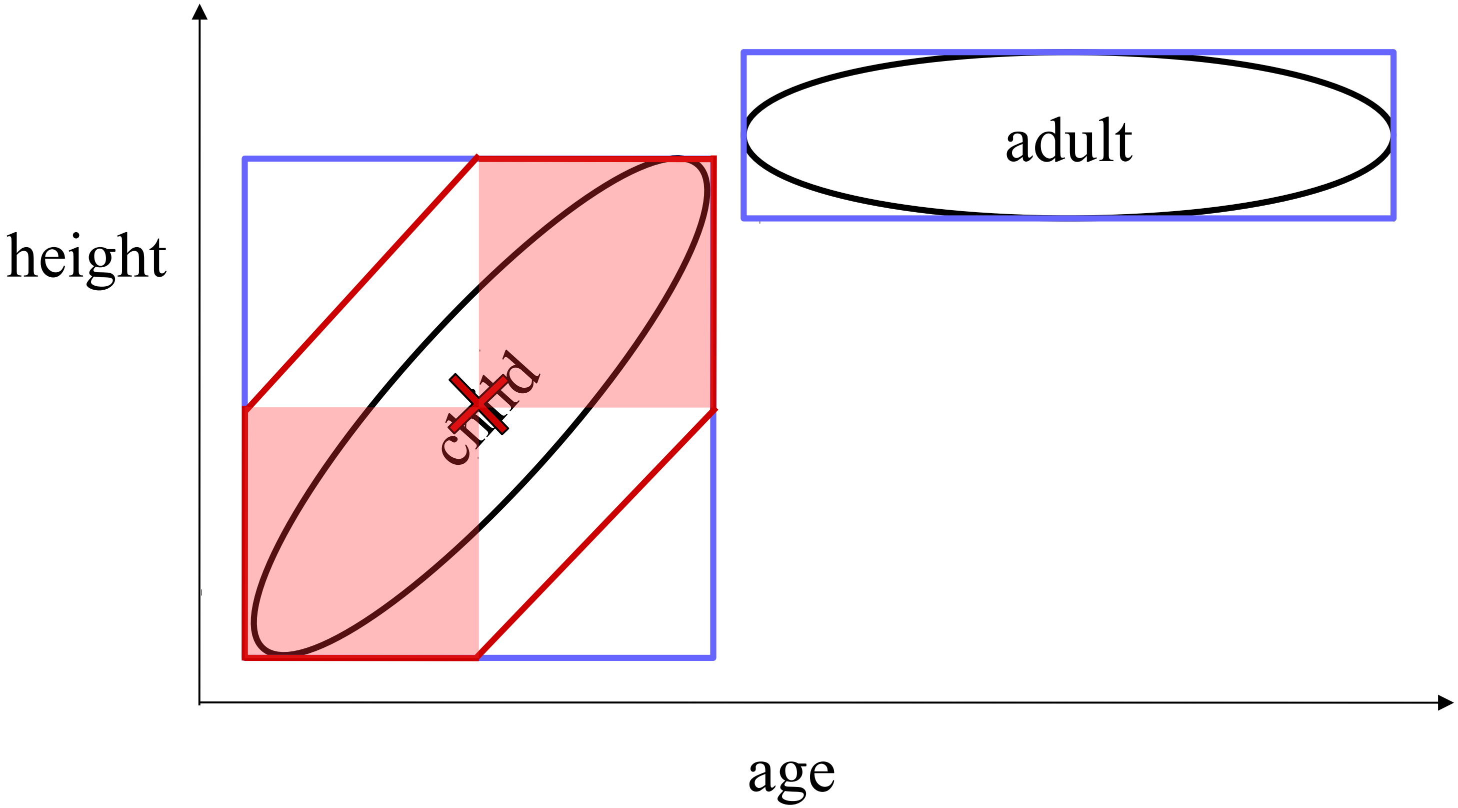
Convexity and Manhattan distance



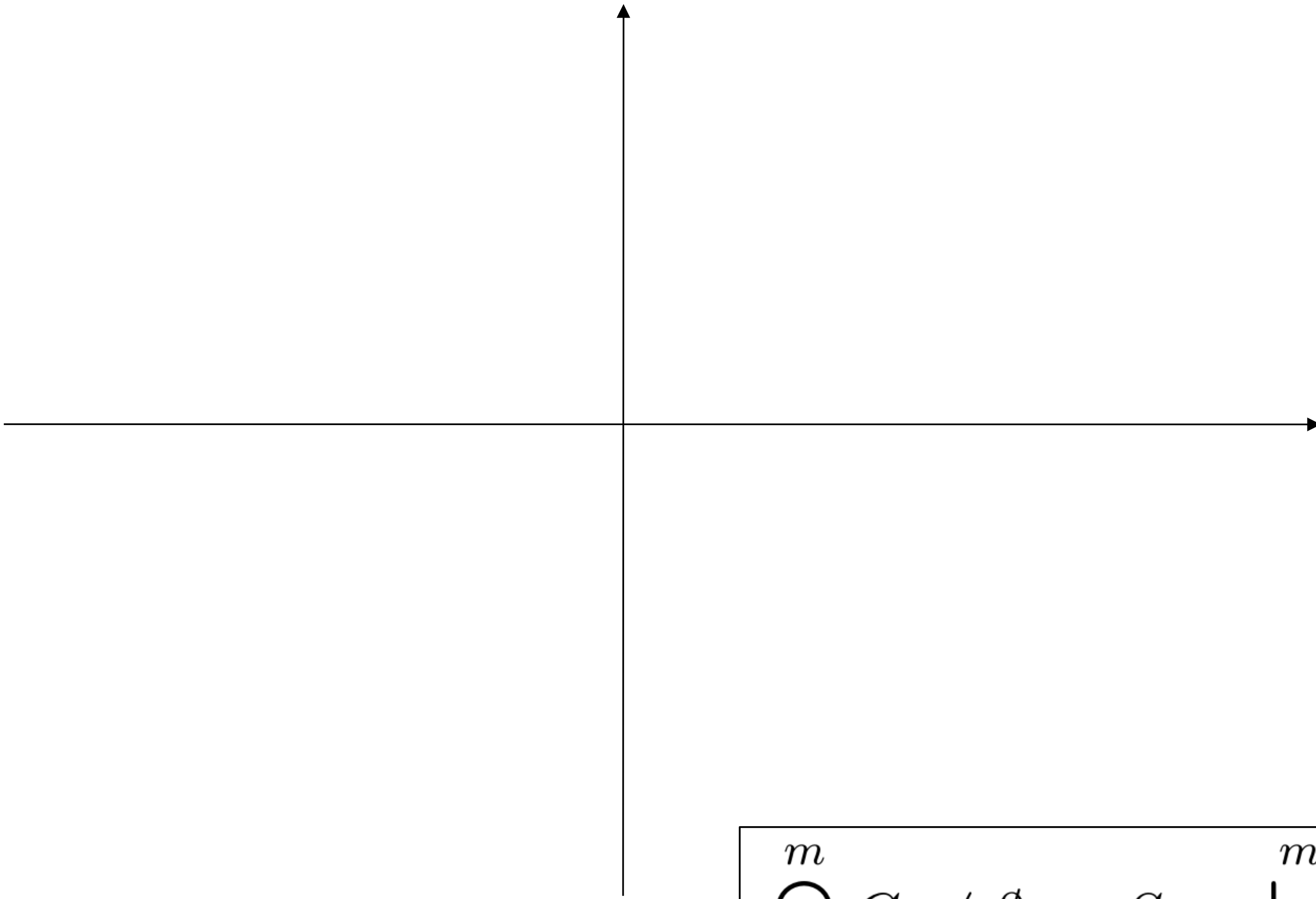
Convexity and Manhattan distance



Convexity and Manhattan distance

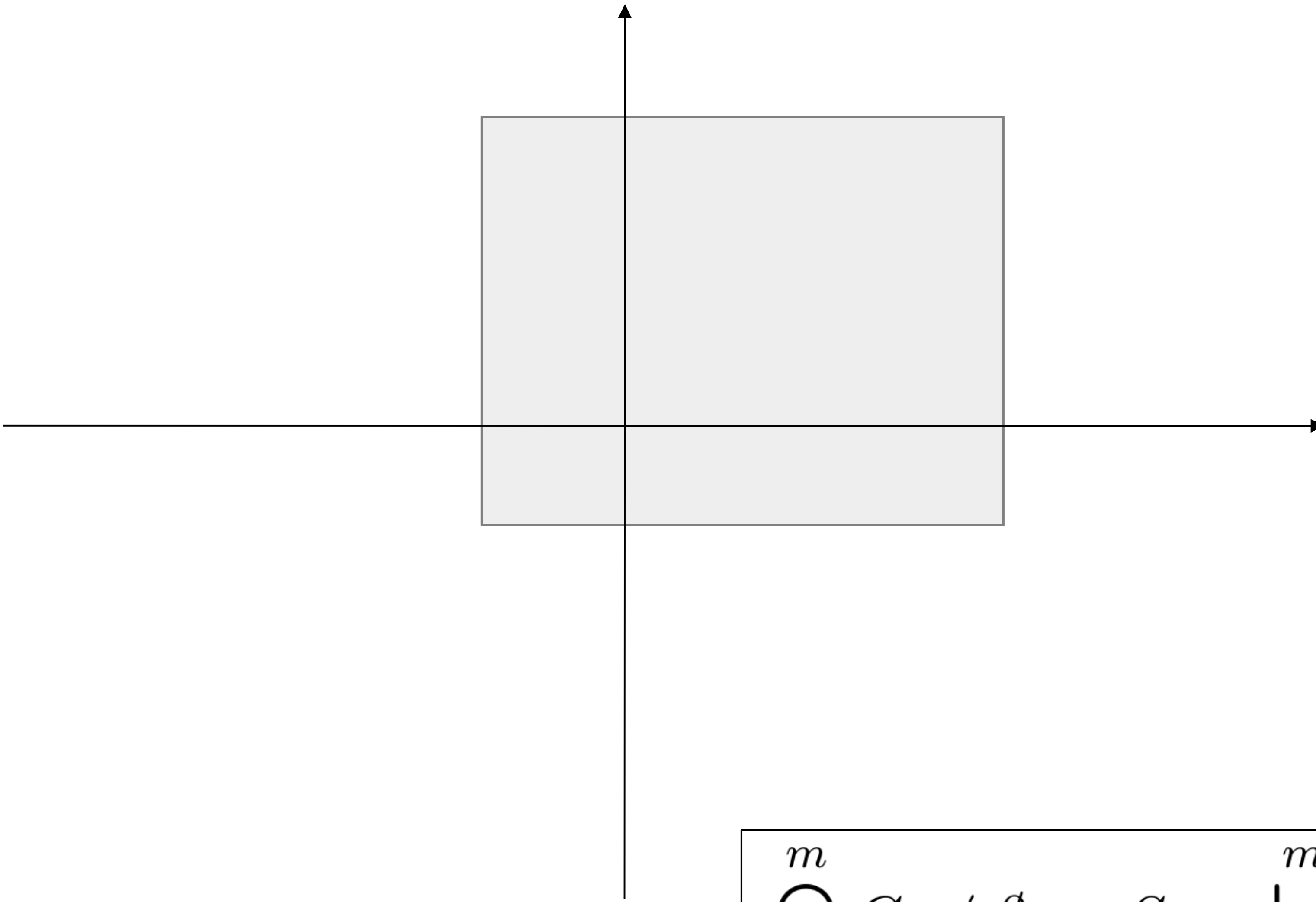


Formalizing Star-Shaped Concepts



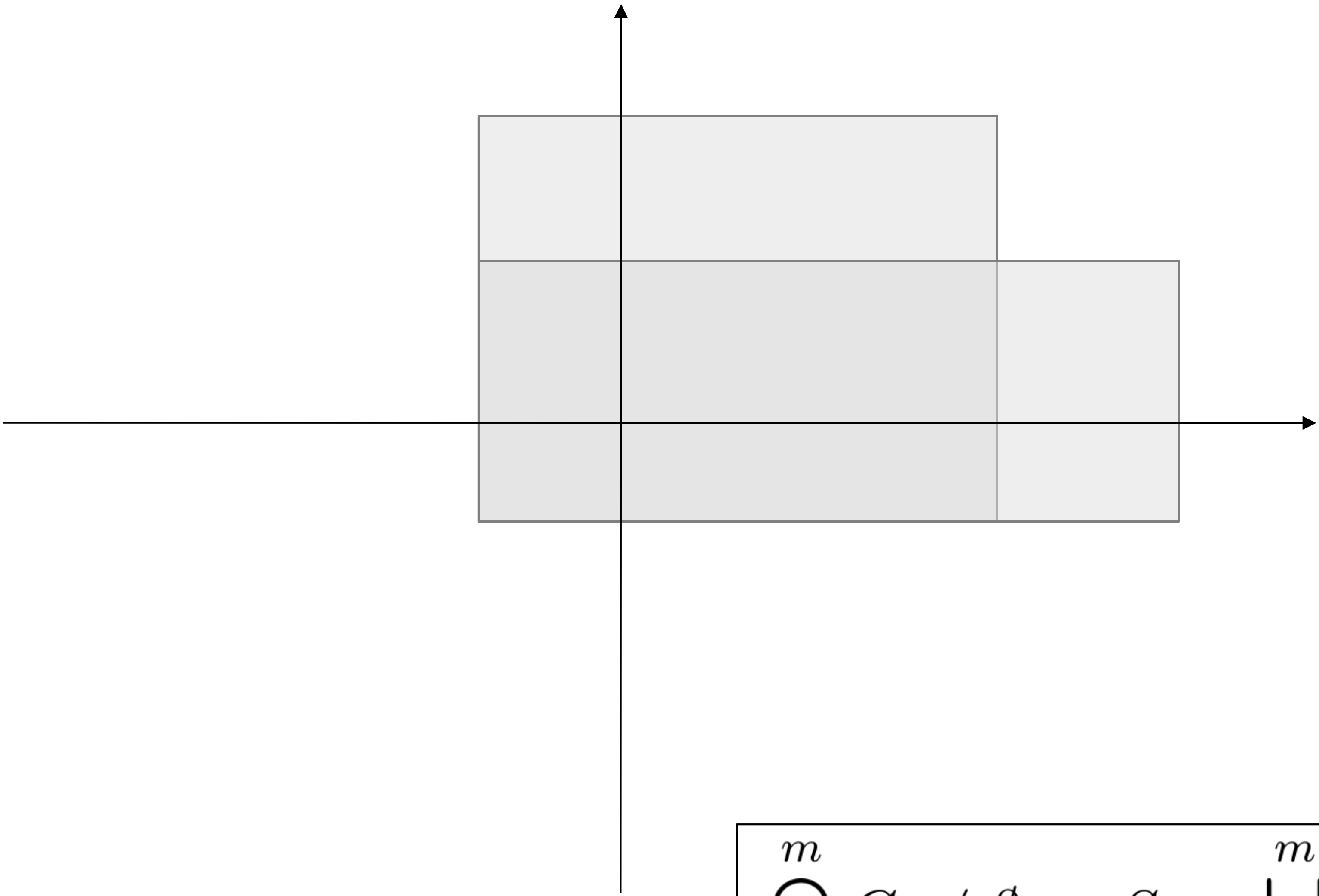
$$\bigcap_{i=1}^{m} C_i \neq \emptyset \Rightarrow S := \bigcup_{i=1}^{m} C_i$$

Formalizing Star-Shaped Concepts



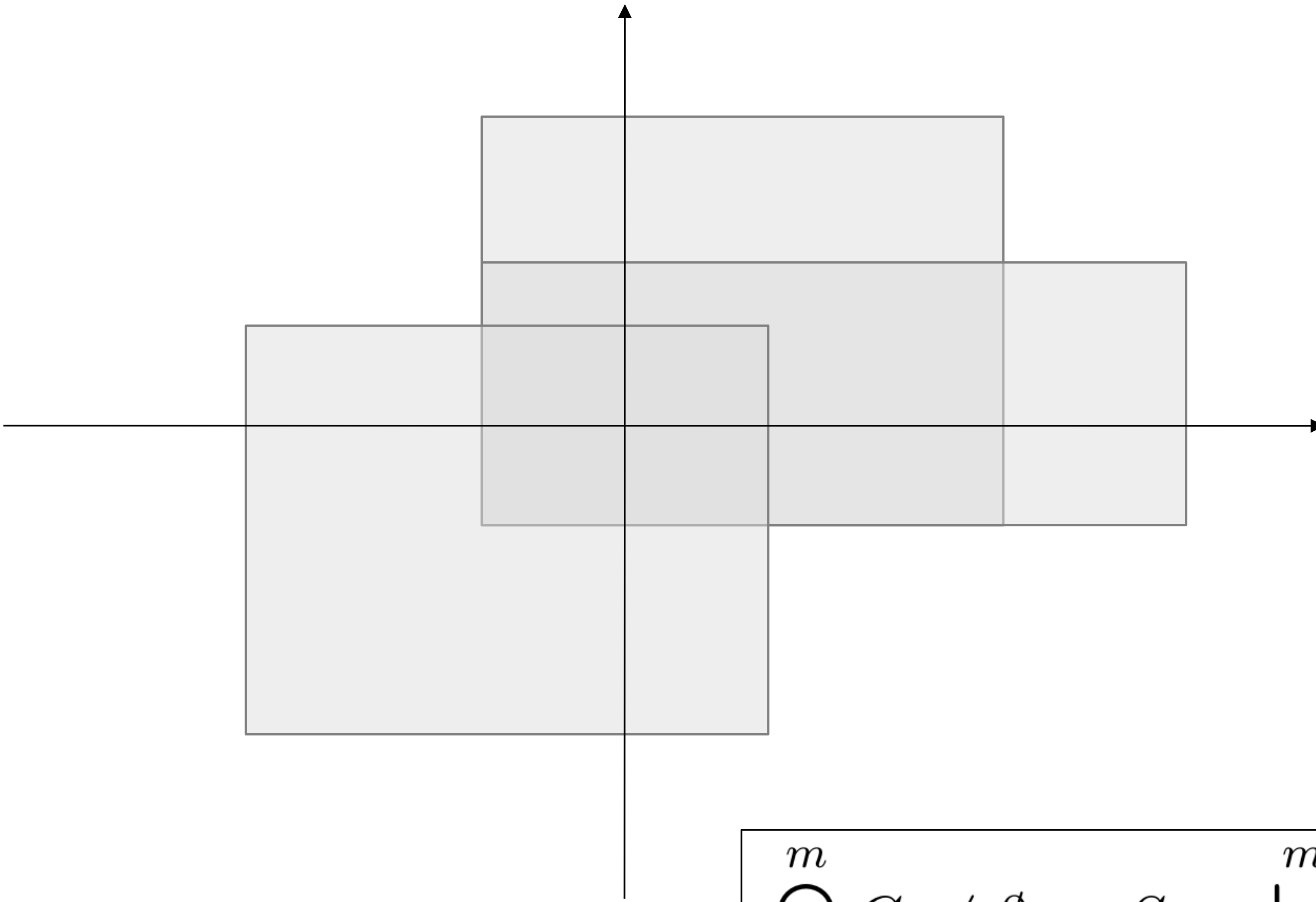
$$\bigcap_{i=1}^{m} C_i \neq \emptyset \Rightarrow S := \bigcup_{i=1}^{m} C_i$$

Formalizing Star-Shaped Concepts



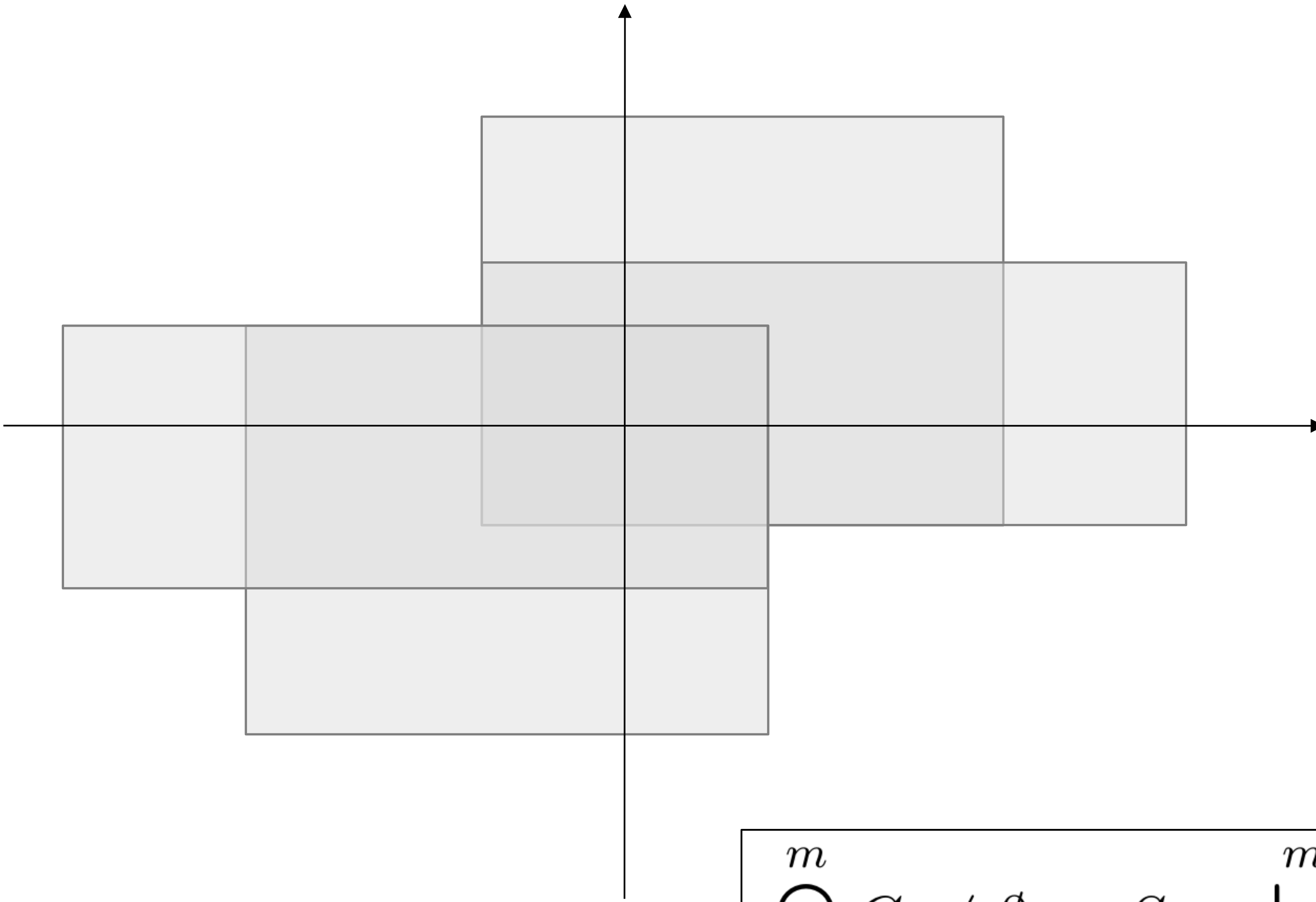
$$\bigcap_{i=1}^{m} C_i \neq \emptyset \Rightarrow S := \bigcup_{i=1}^{m} C_i$$

Formalizing Star-Shaped Concepts



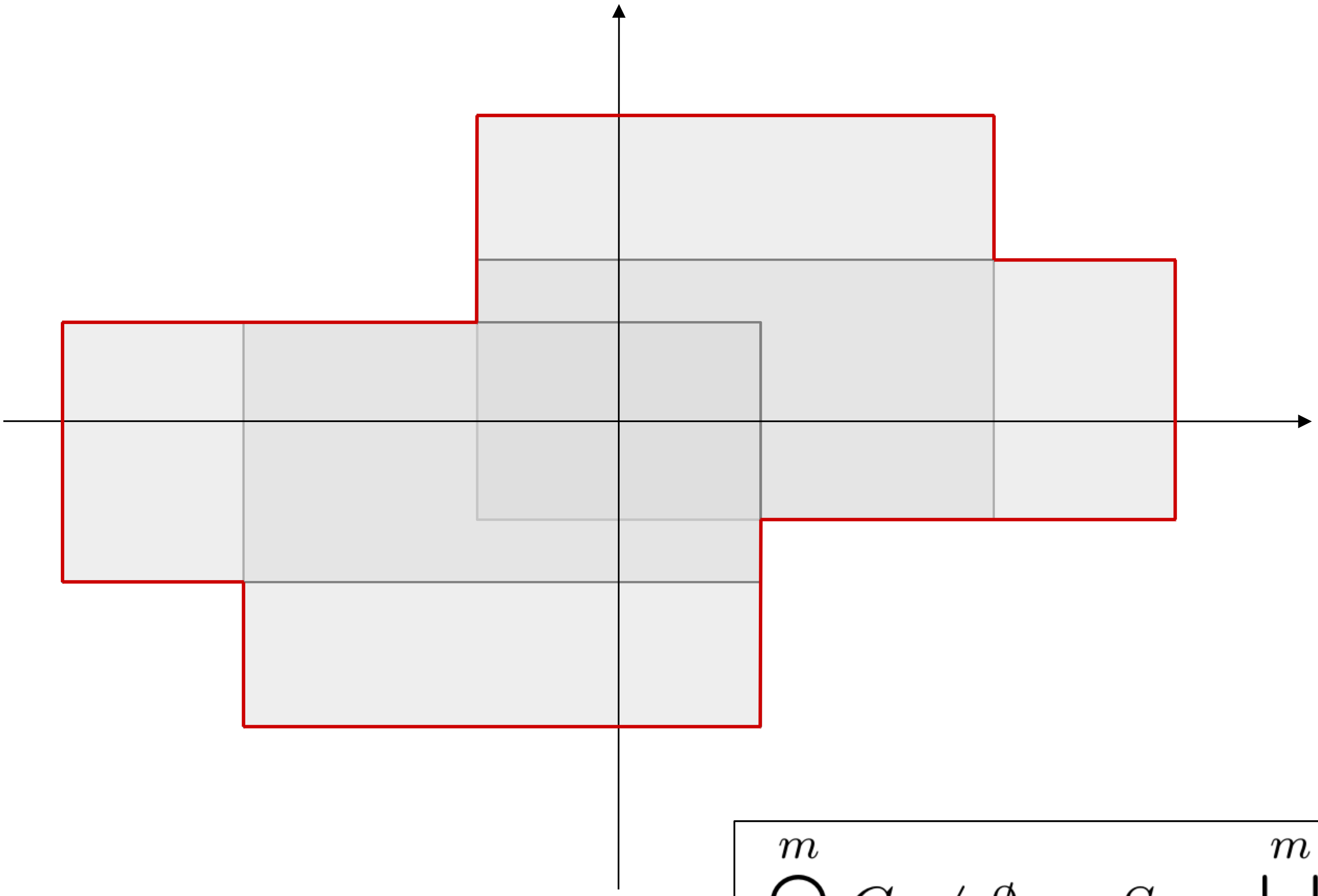
$$\bigcap_{i=1}^{m} C_i \neq \emptyset \Rightarrow S := \bigcup_{i=1}^{m} C_i$$

Formalizing Star-Shaped Concepts



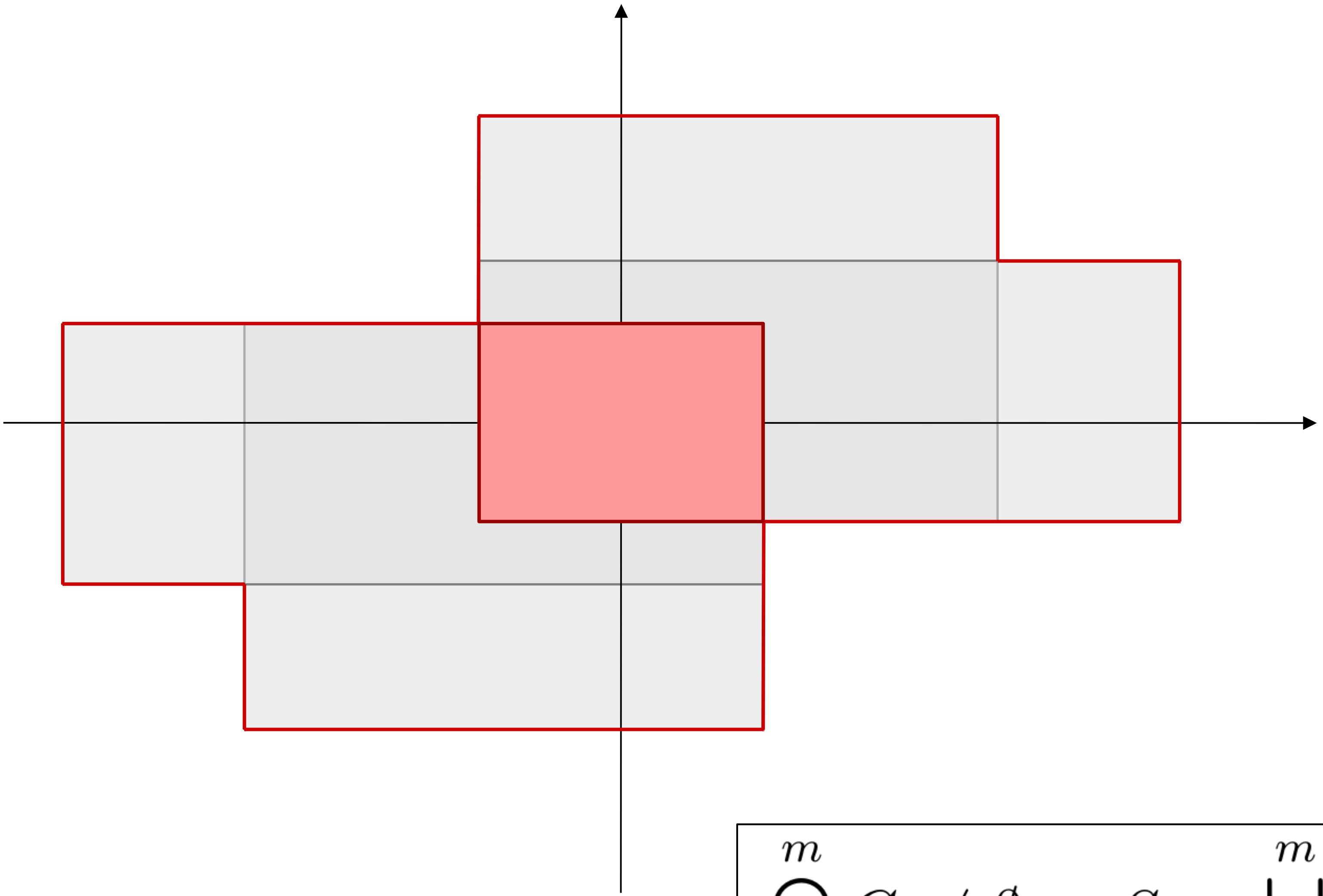
$$\bigcap_{i=1}^{m} C_i \neq \emptyset \Rightarrow S := \bigcup_{i=1}^{m} C_i$$

Formalizing Star-Shaped Concepts



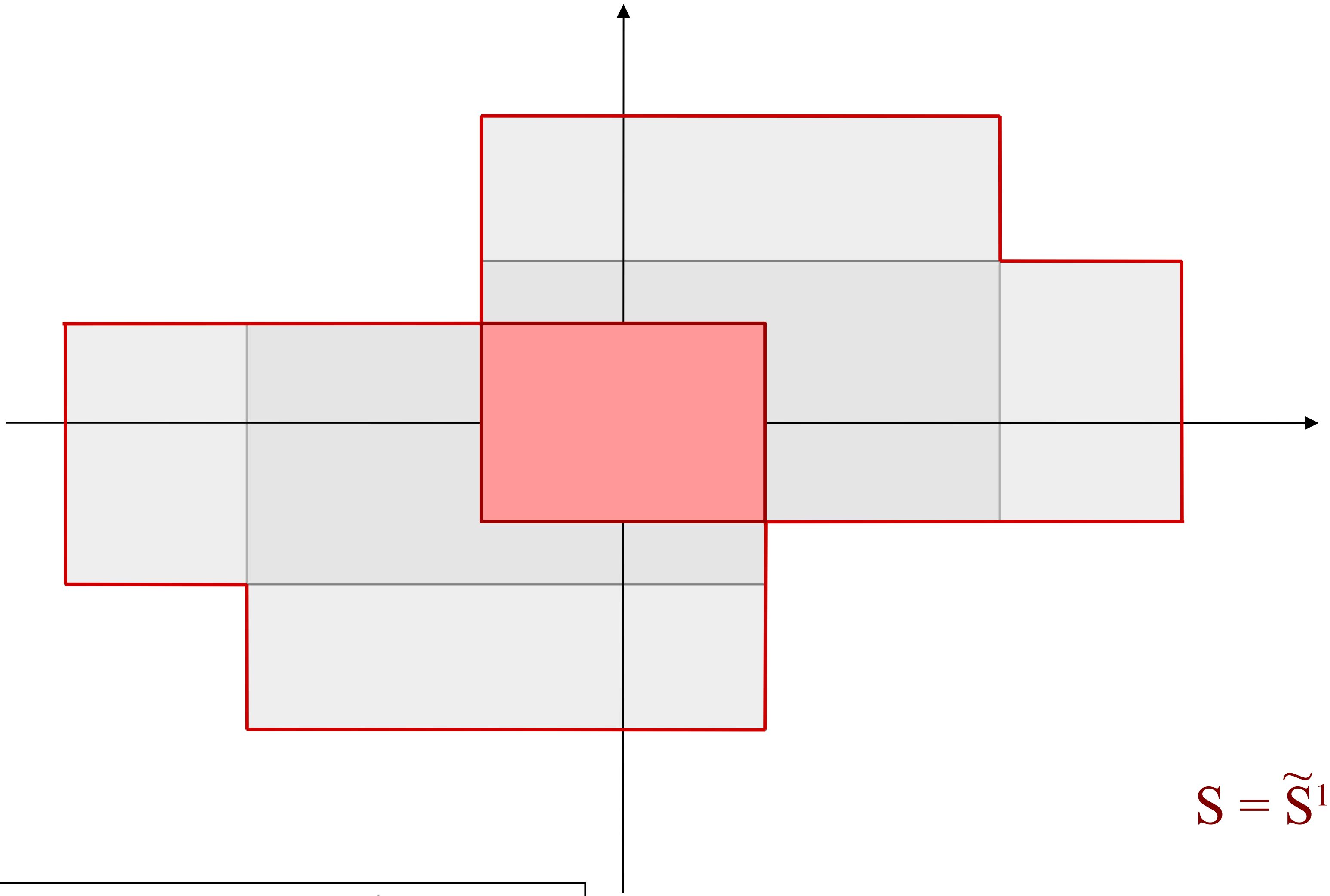
$$\bigcap_{i=1}^{m} C_i \neq \emptyset \Rightarrow S := \bigcup_{i=1}^{m} C_i$$

Formalizing Star-Shaped Concepts



$$\bigcap_{i=1}^{m} C_i \neq \emptyset \Rightarrow S := \bigcup_{i=1}^{m} C_i$$

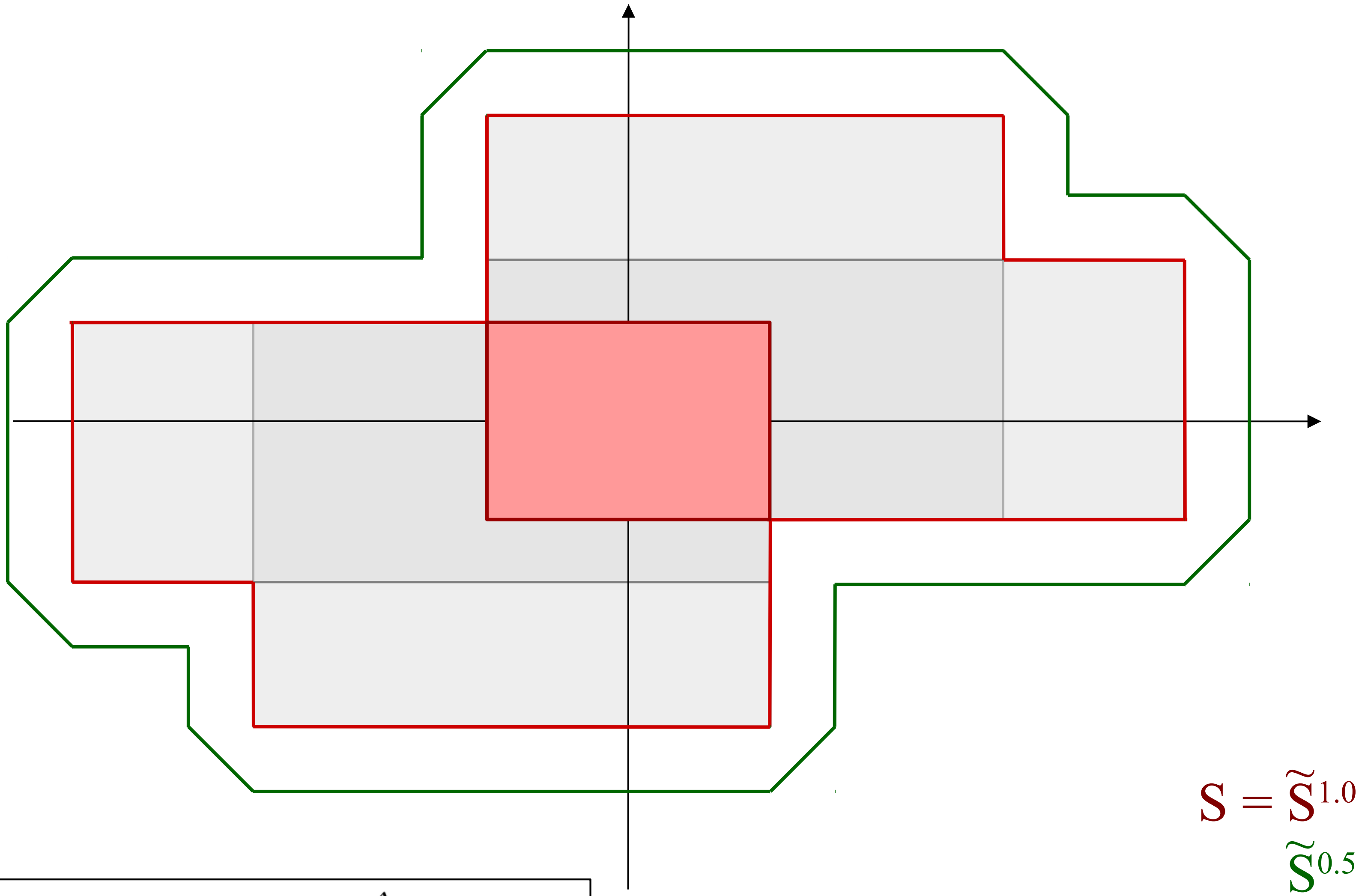
Formalizing Star-Shaped Concepts



$S = \widetilde{S}^{1.0}$

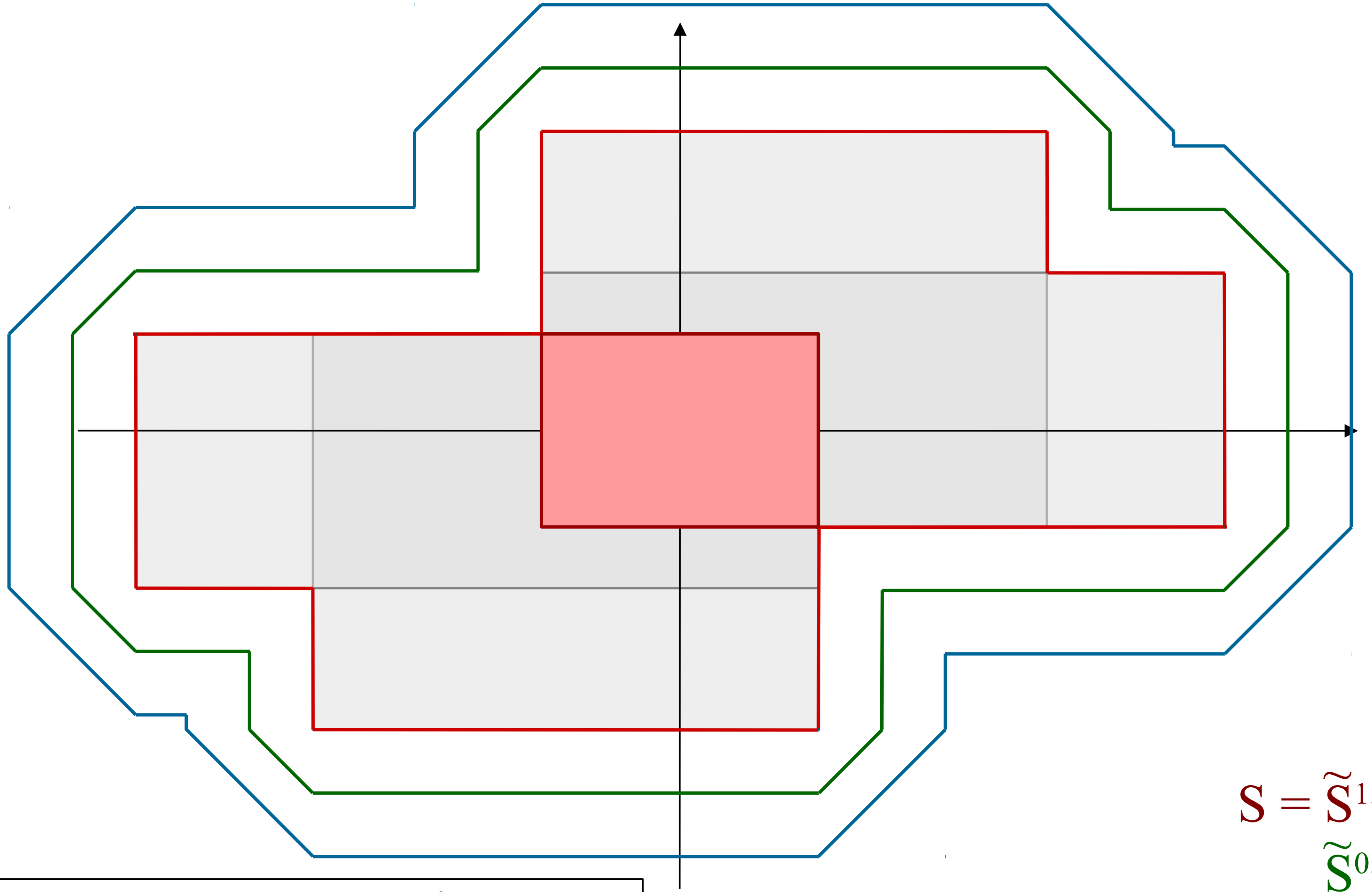
$$\mu_{\widetilde{S}}(x) = \mu_0 \cdot \max_{y \in S}\left(e^{-c \cdot d_C^{\Delta_S}(x,y,W)}\right)$$

Formalizing Star-Shaped Concepts



$$\mu_{\widetilde{S}}(x) = \mu_0 \cdot \max_{y \in S}(e^{-c \cdot d_C^{\Delta_S}(x,y,W)})$$

Formalizing Star-Shaped Concepts



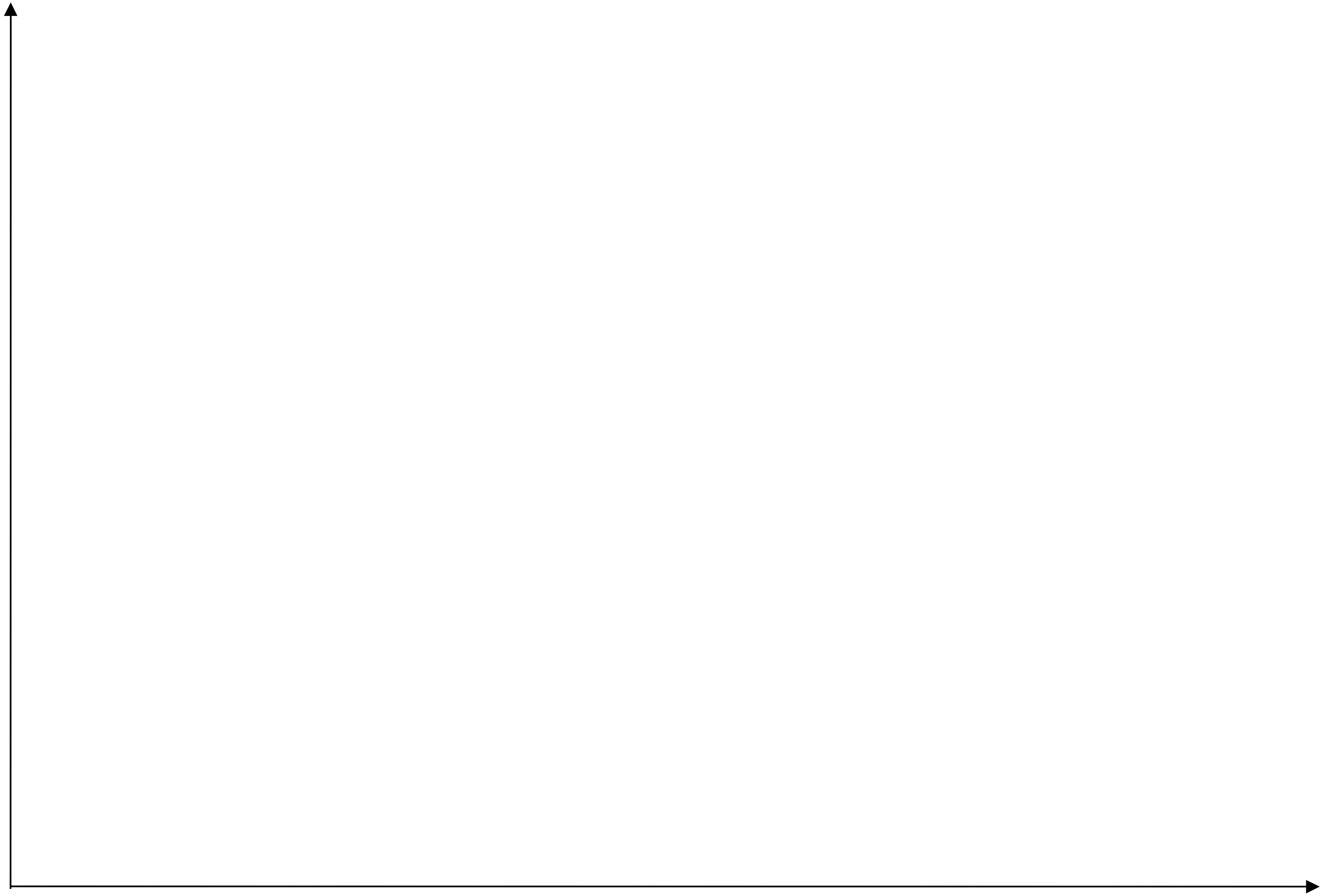
$$\mu_{\widetilde{S}}(x) = \mu_0 \cdot \max_{y \in S}(e^{-c \cdot d_C^{\Delta_S}(x,y,W)})$$

$S = \widetilde{S}^{1.0}$

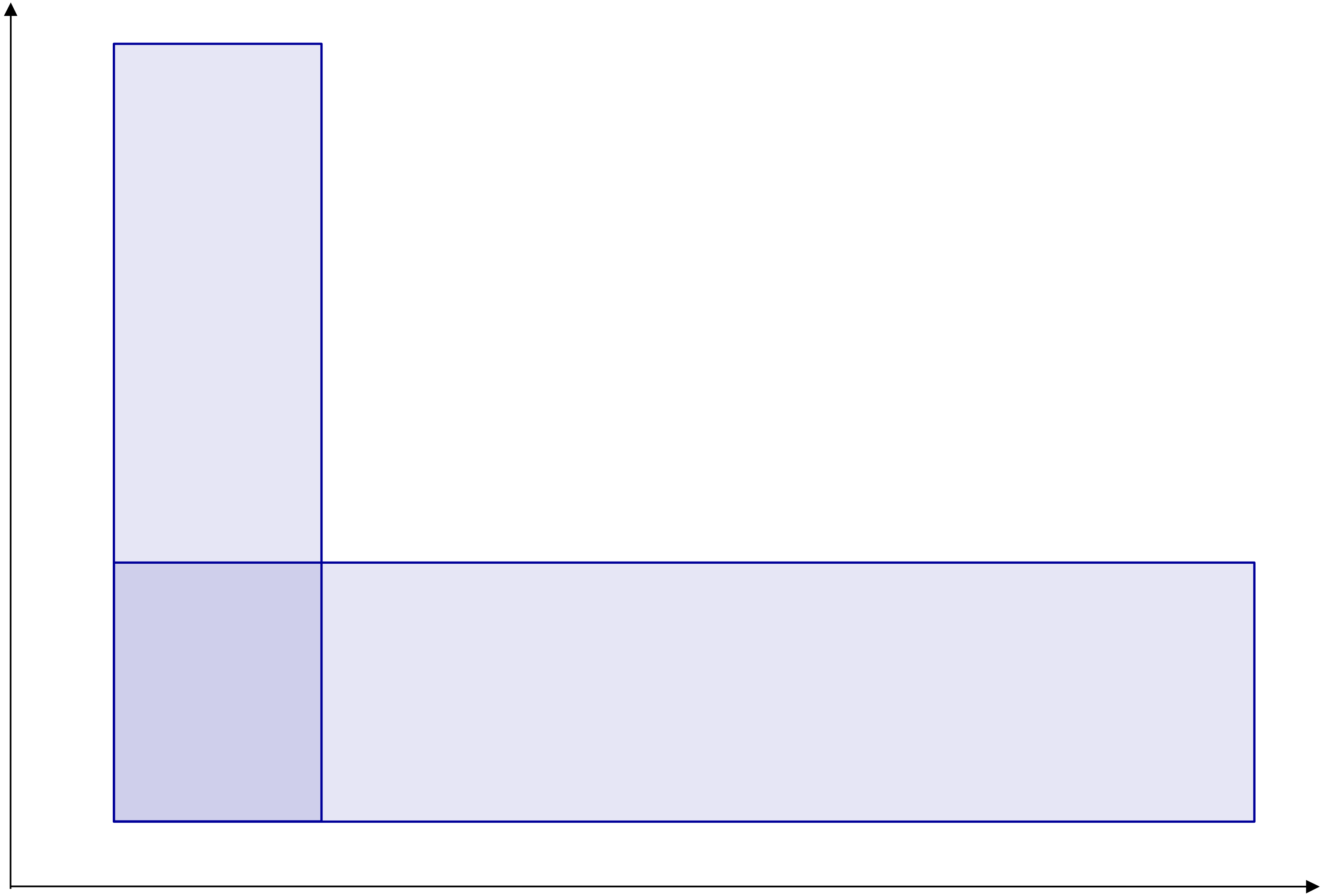
$\widetilde{S}^{0.5}$

$\widetilde{S}^{0.25}$

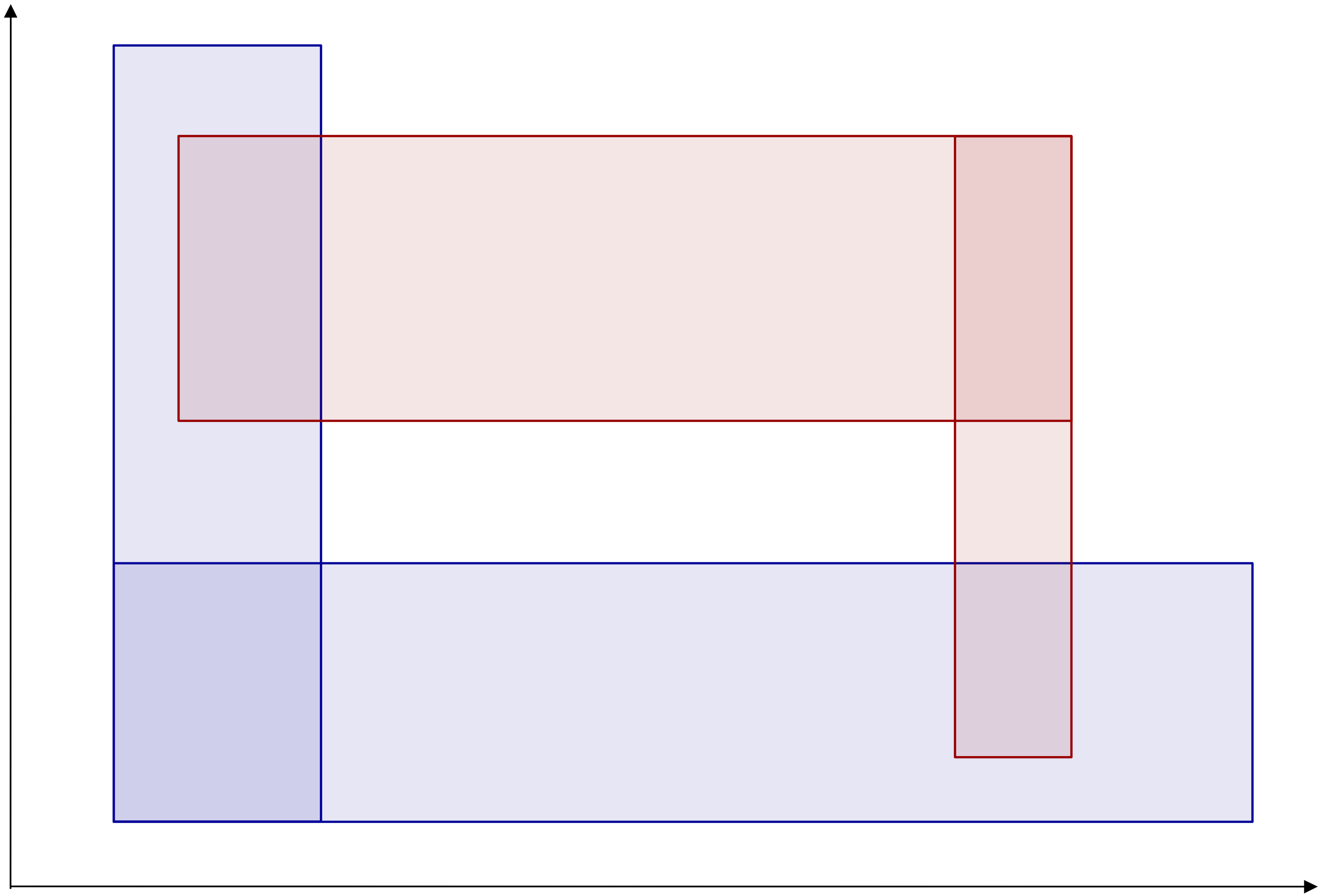
Intersection of Two Concepts



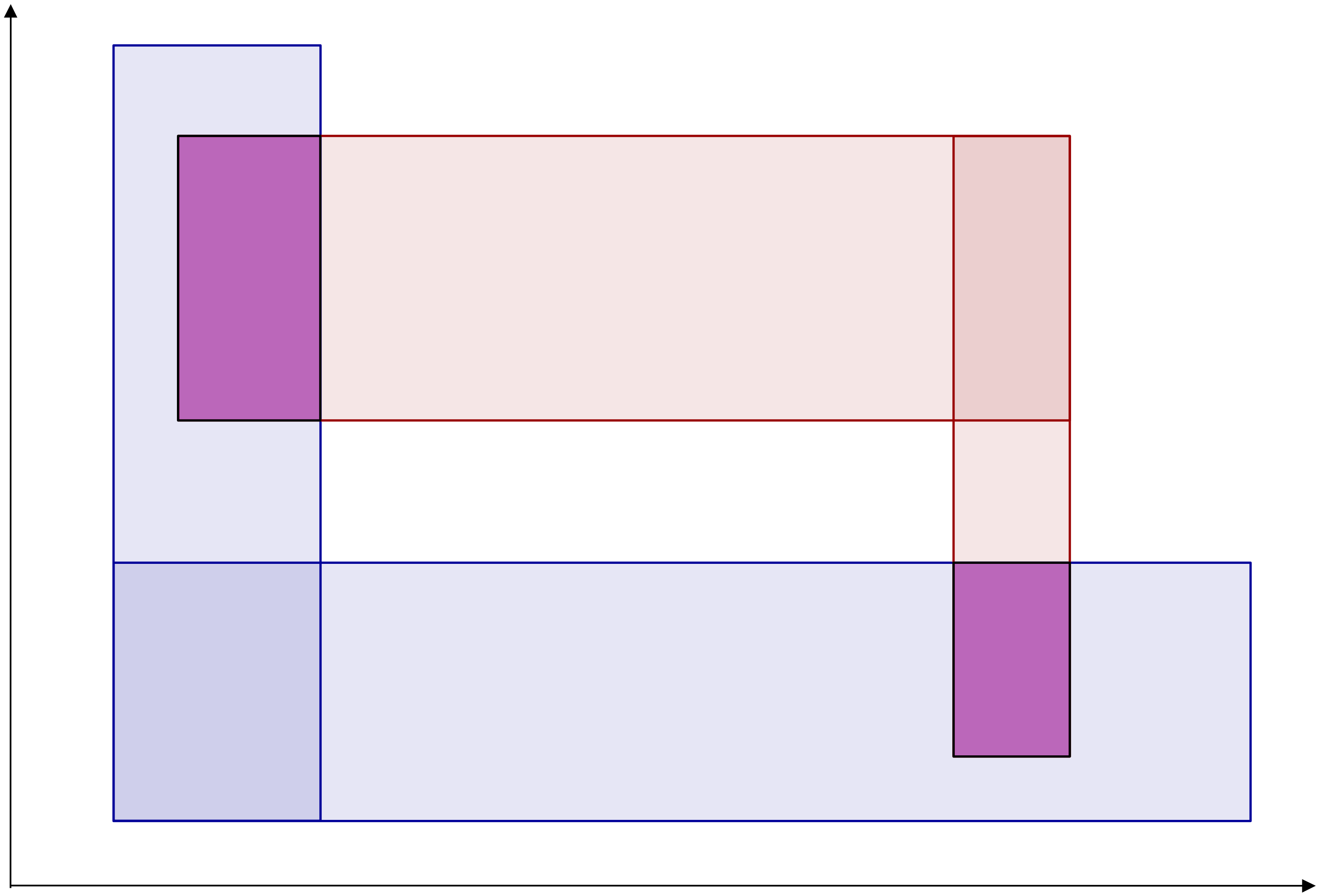
Intersection of Two Concepts



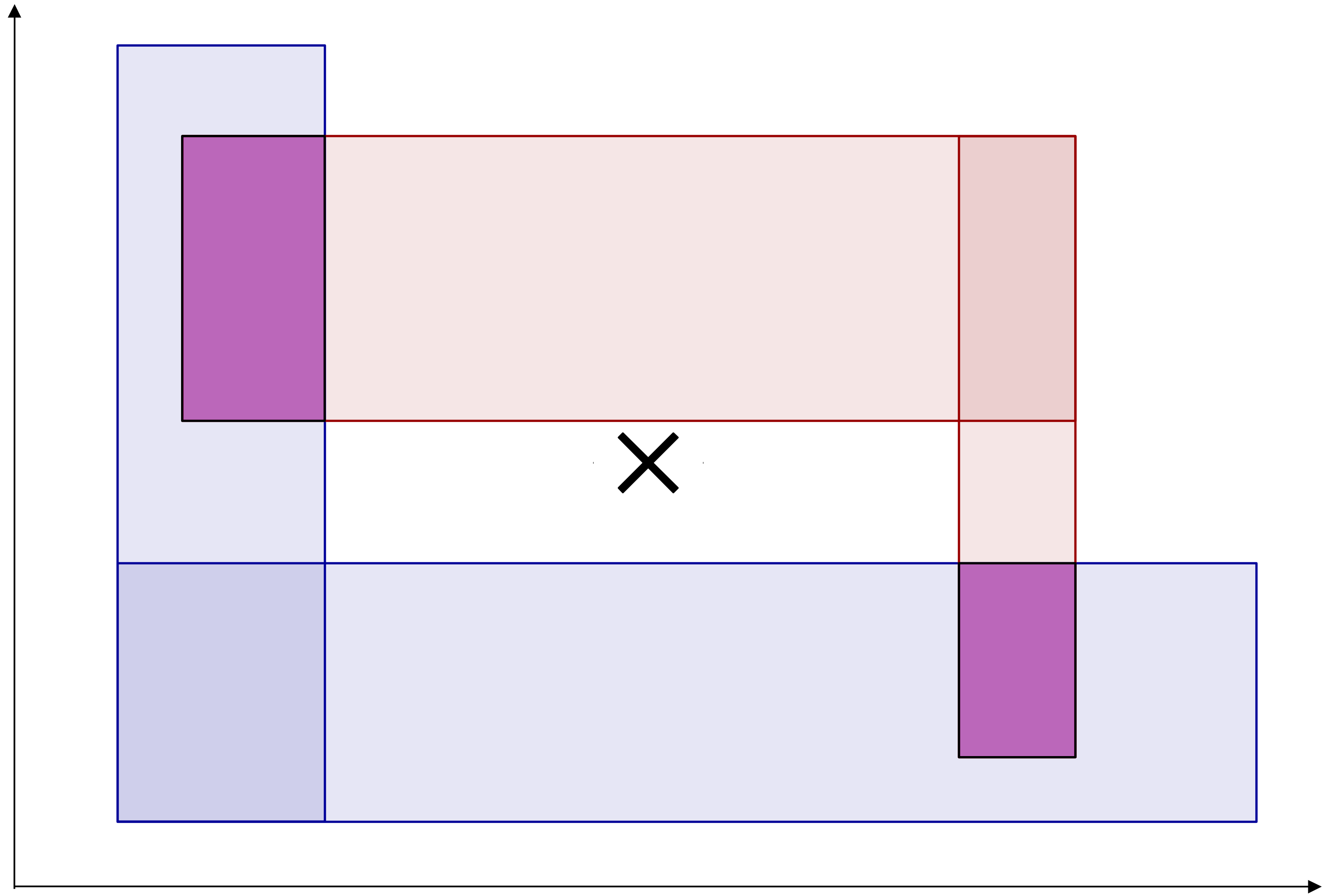
Intersection of Two Concepts



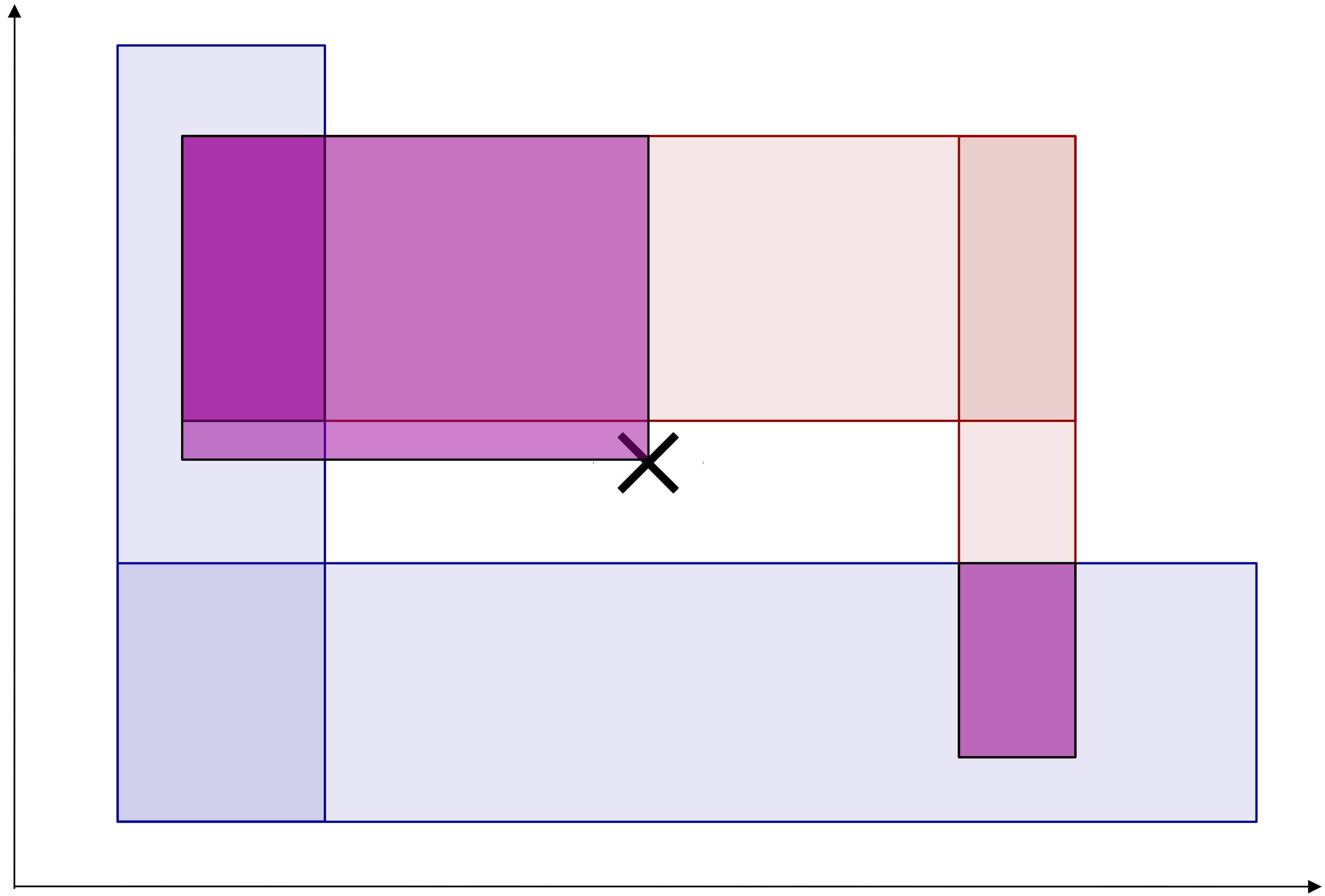
Intersection of Two Concepts



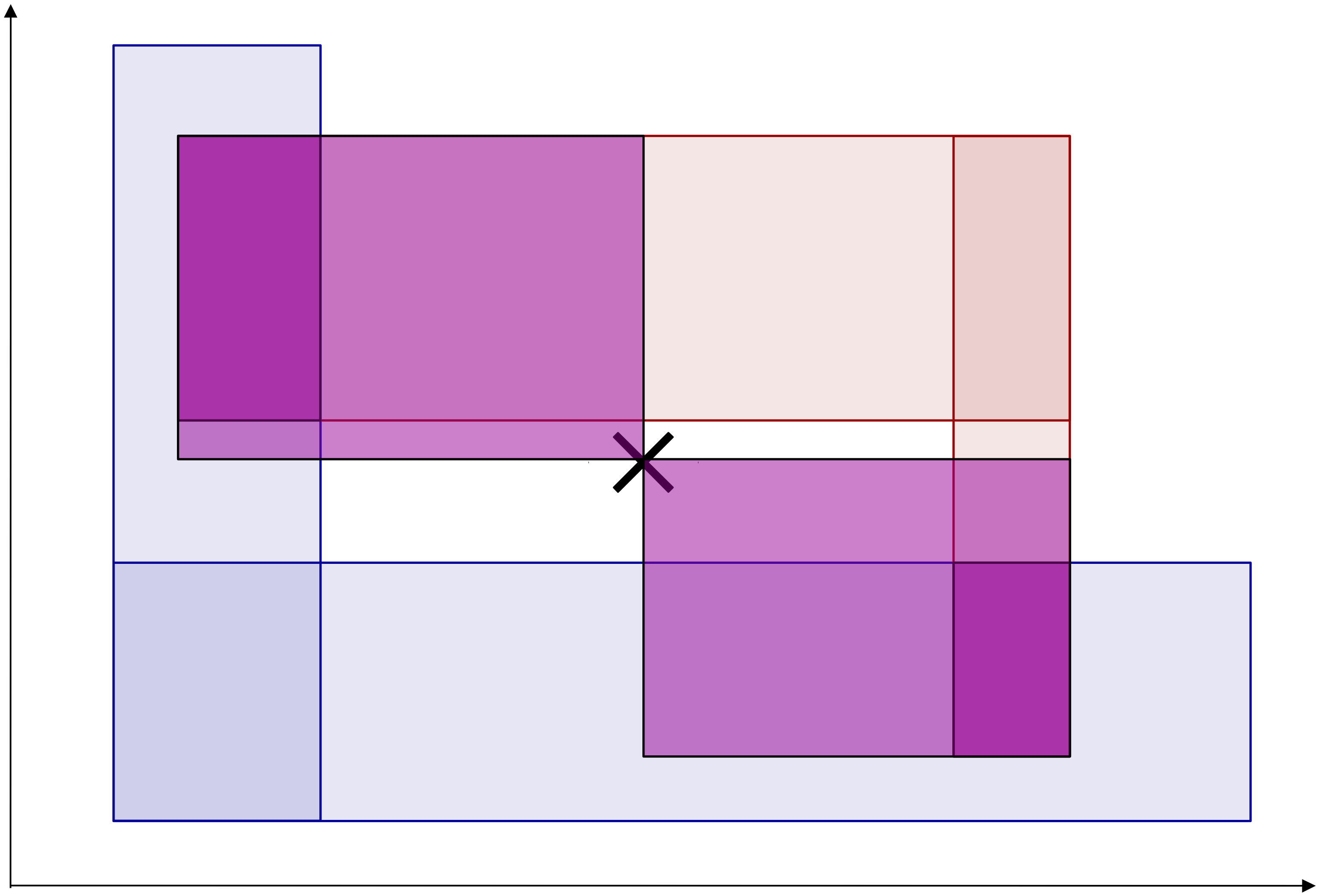
Intersection of Two Concepts



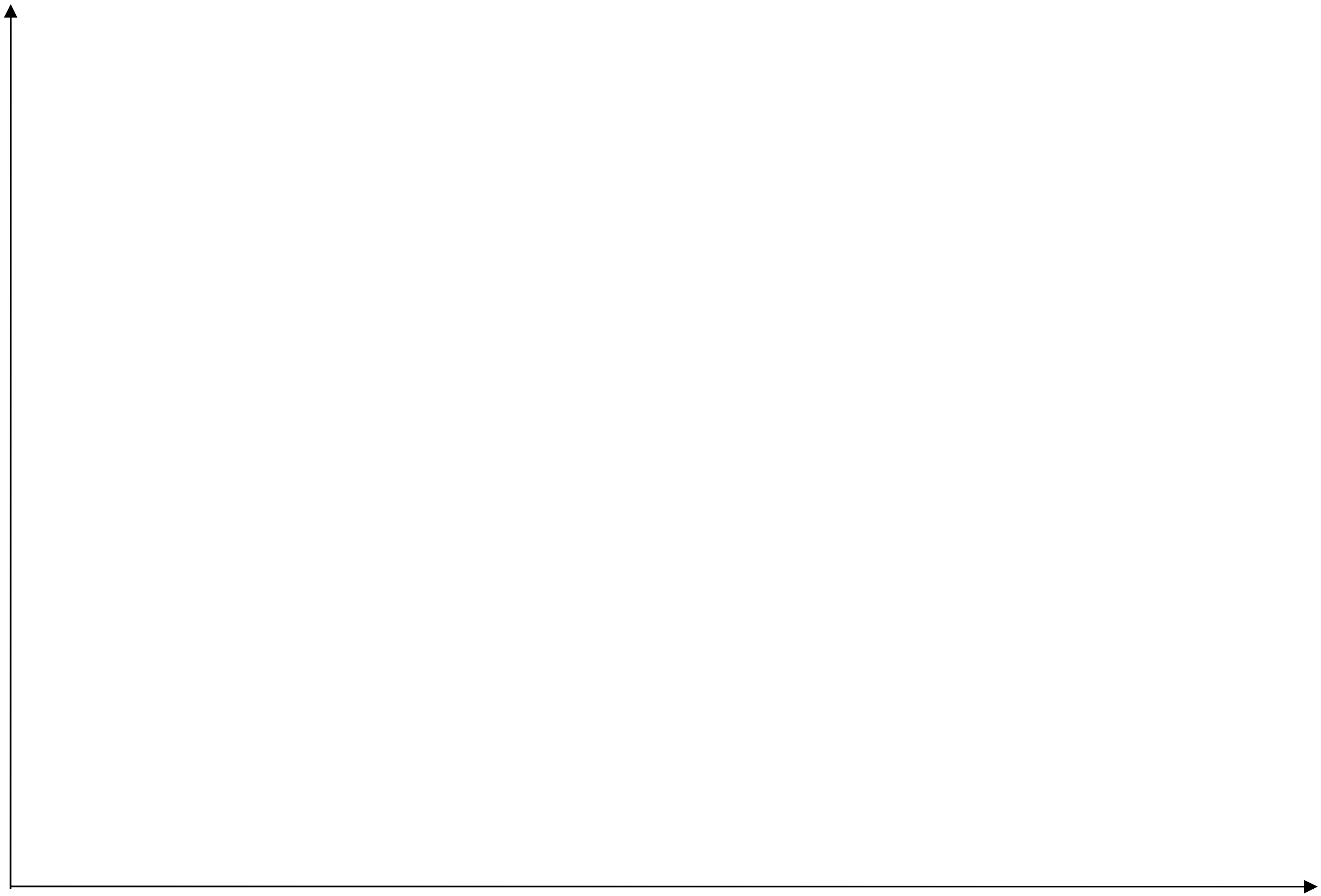
Intersection of Two Concepts



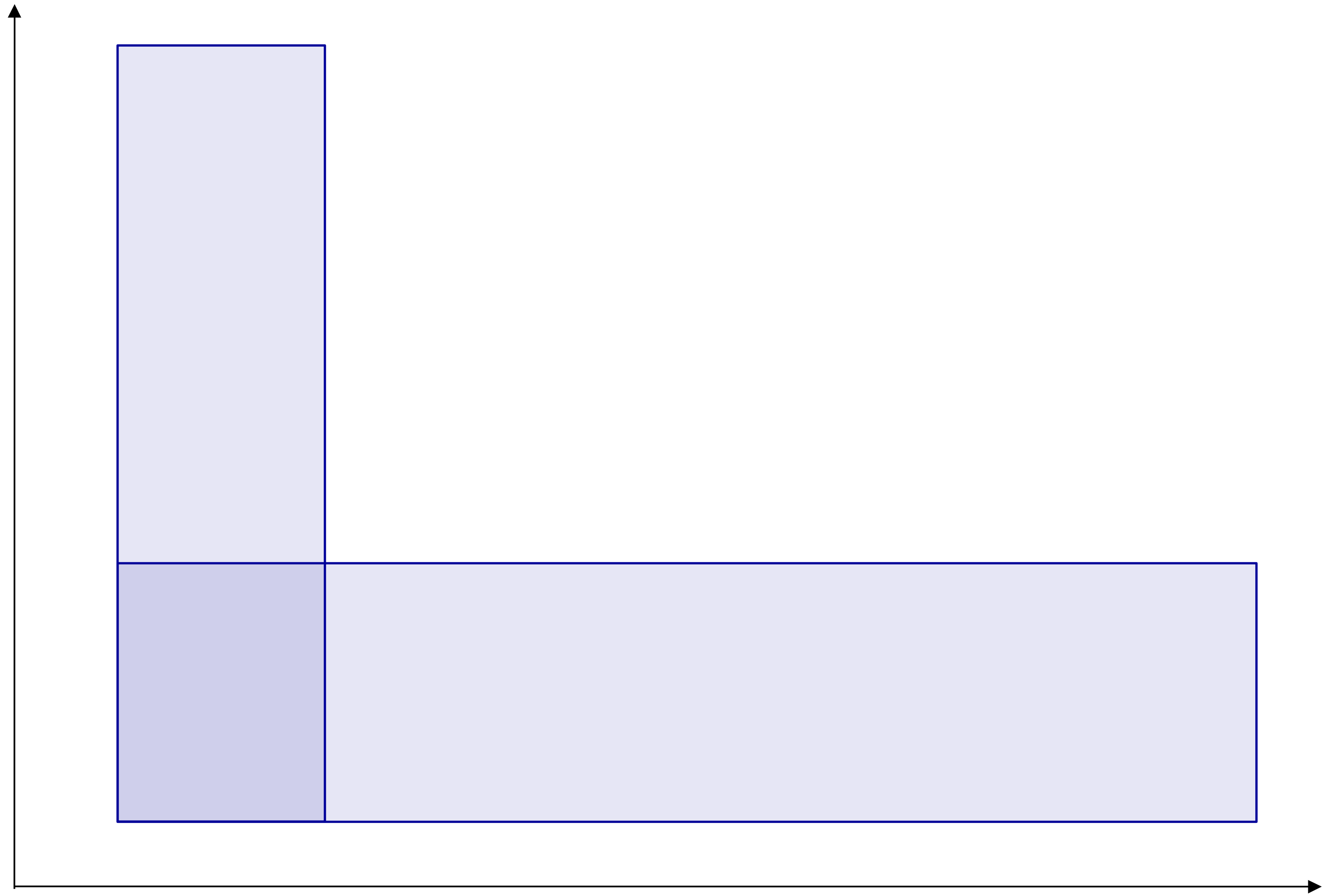
Intersection of Two Concepts



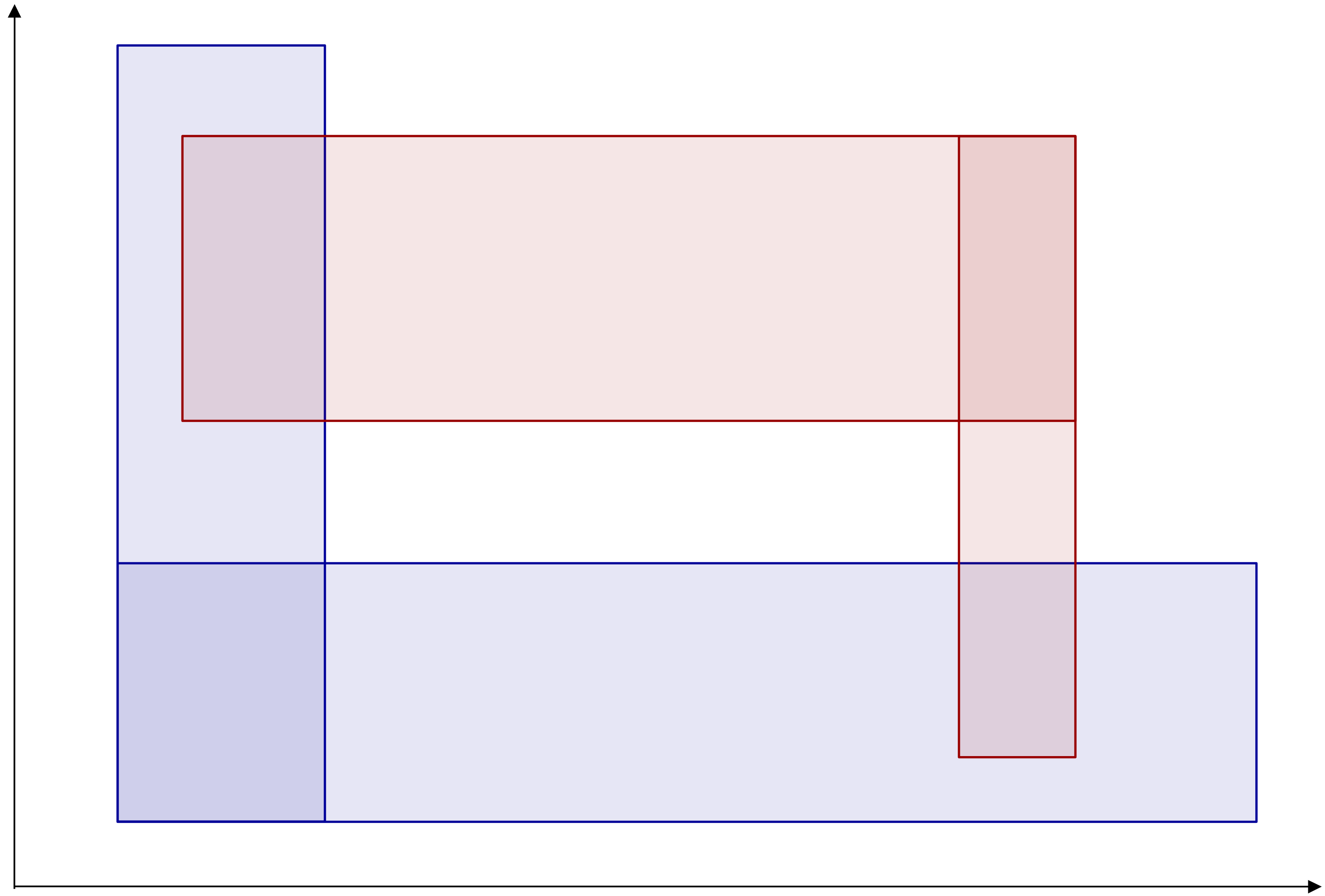
Unification of Two Concepts



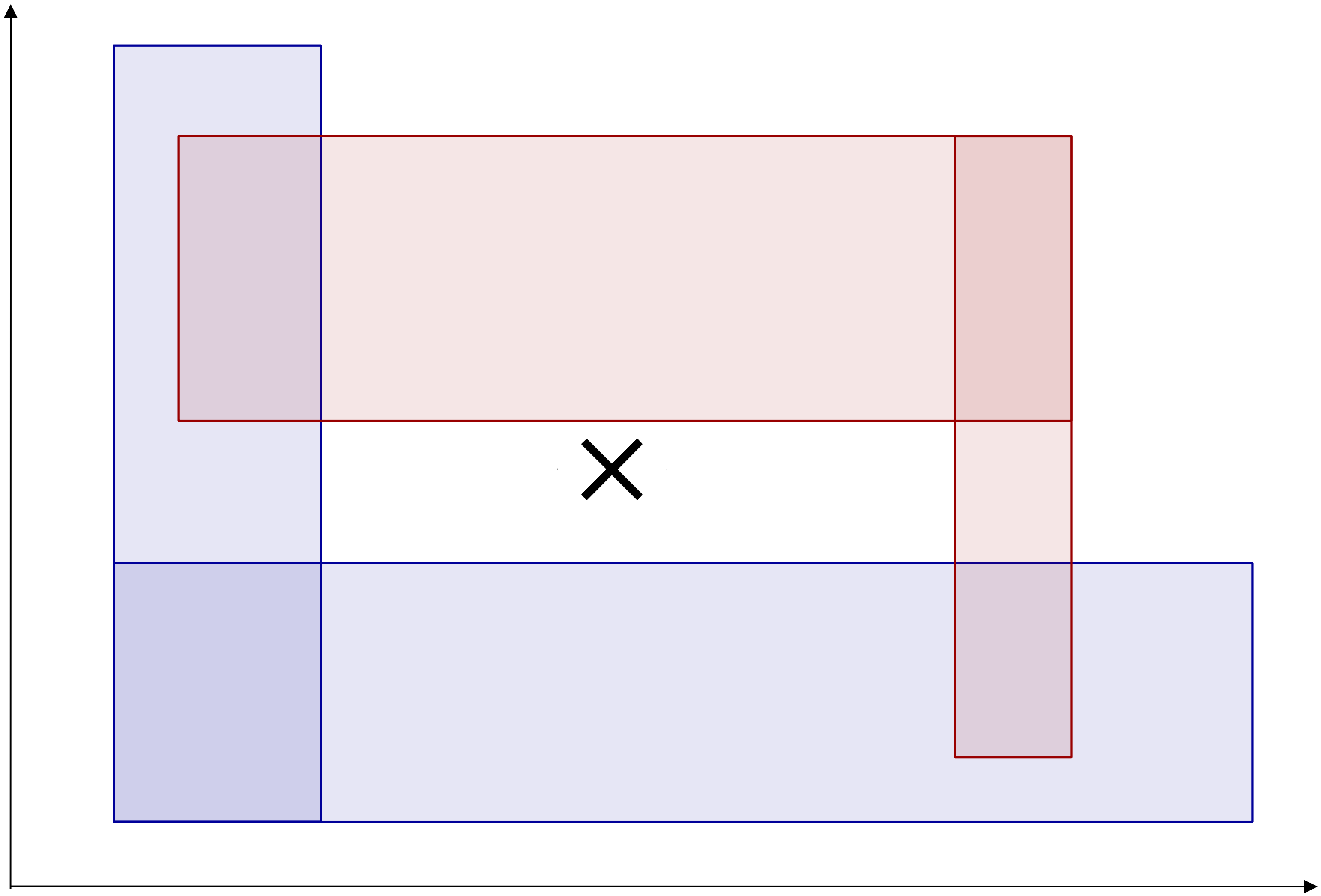
Unification of Two Concepts



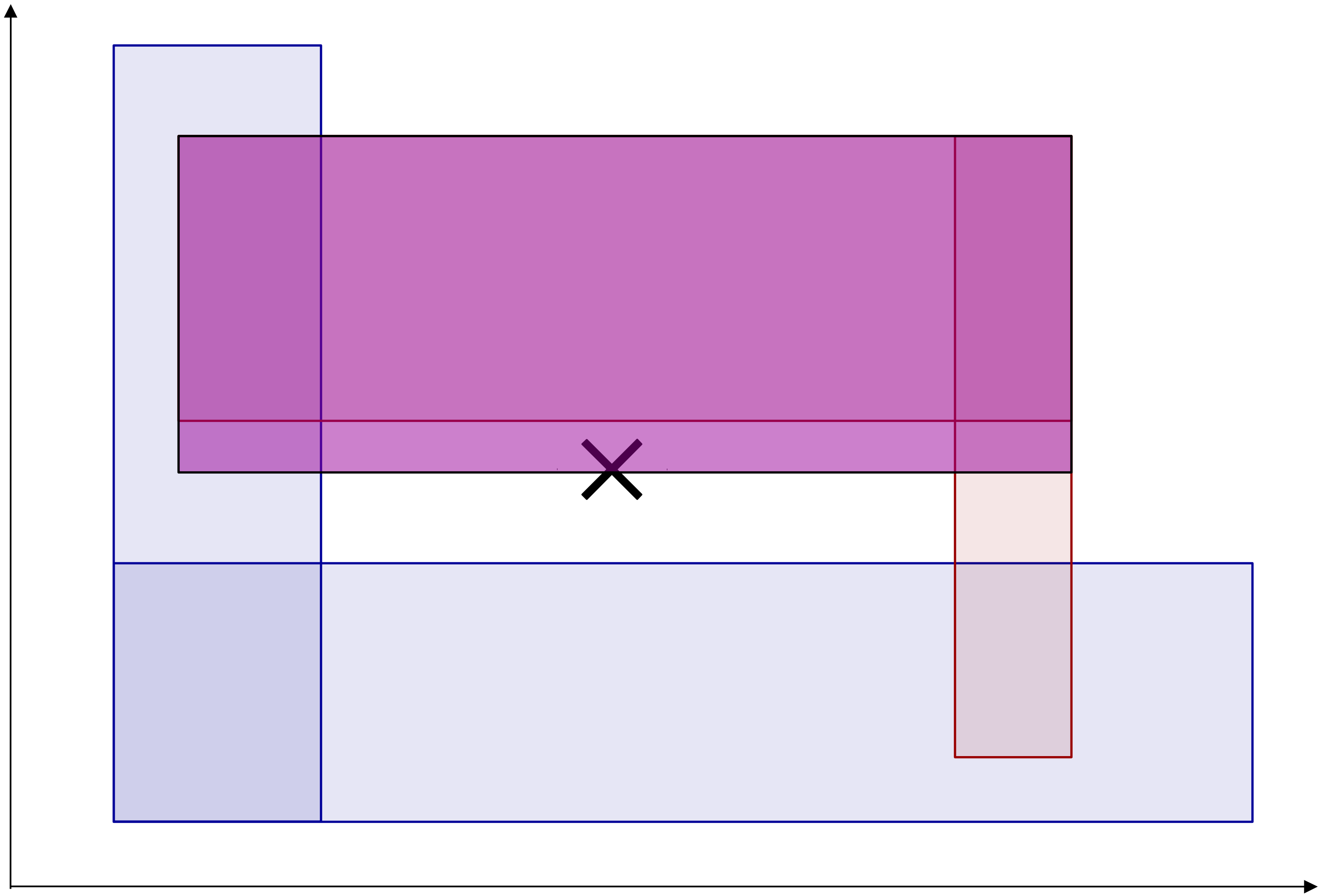
Unification of Two Concepts



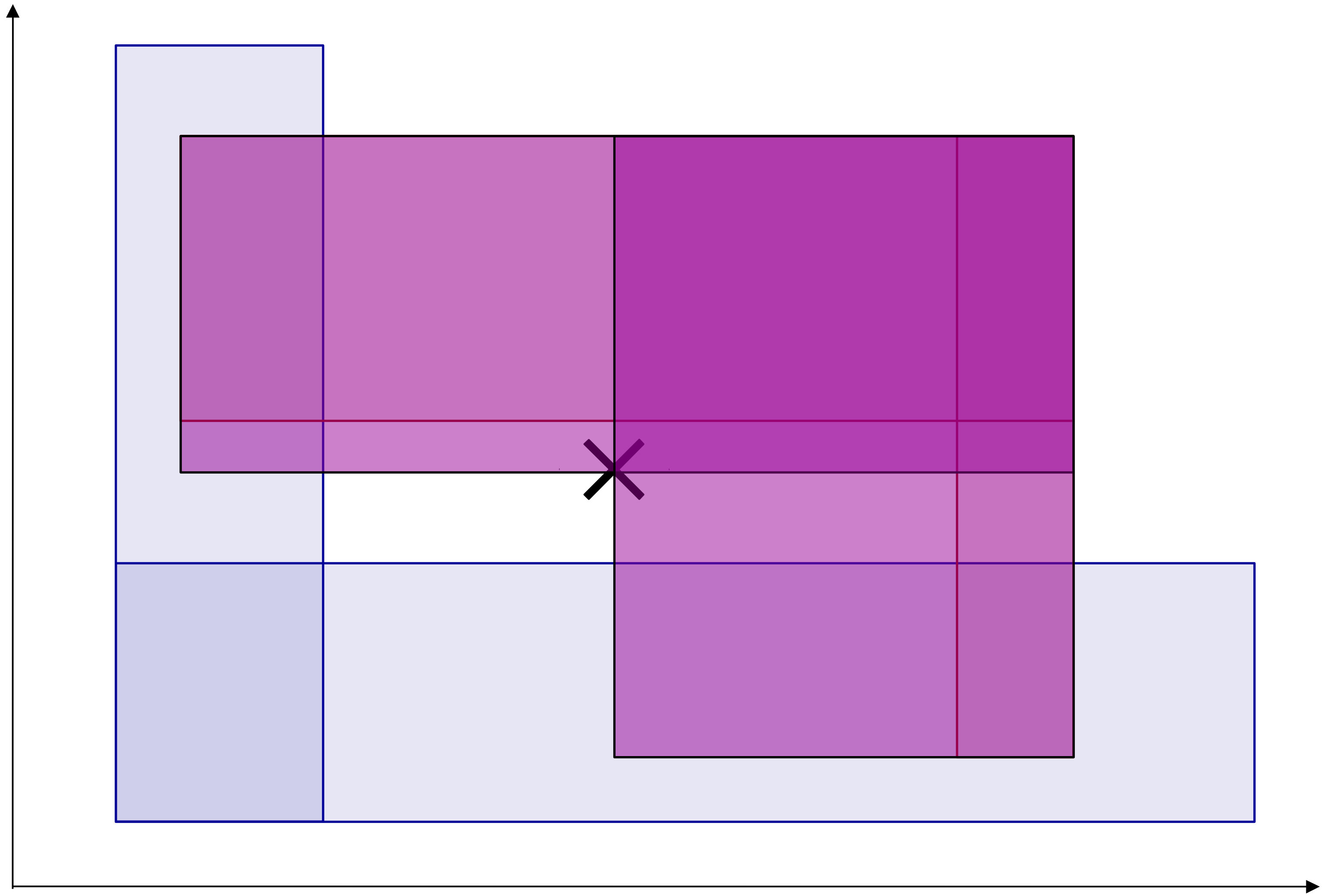
Unification of Two Concepts



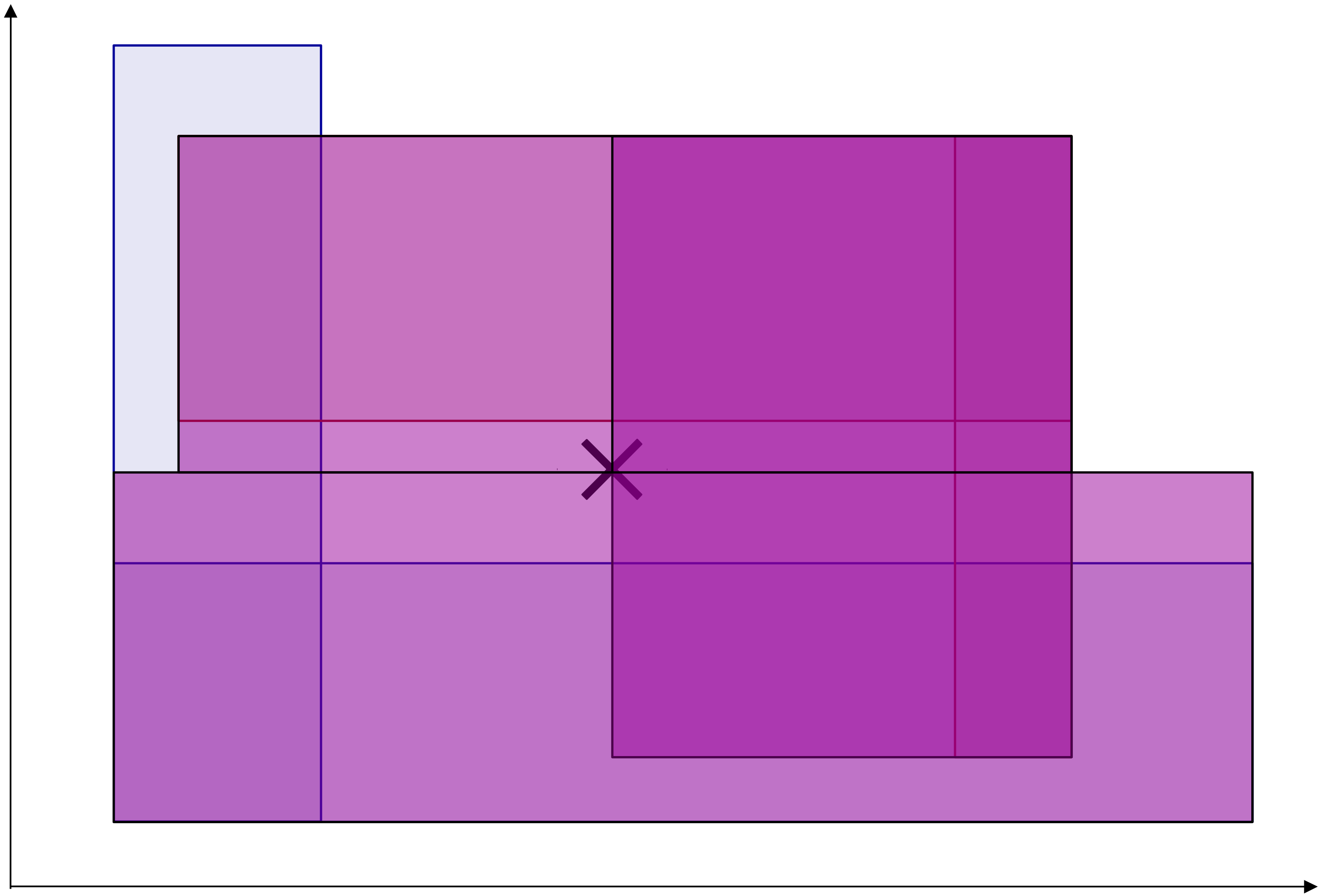
Unification of Two Concepts



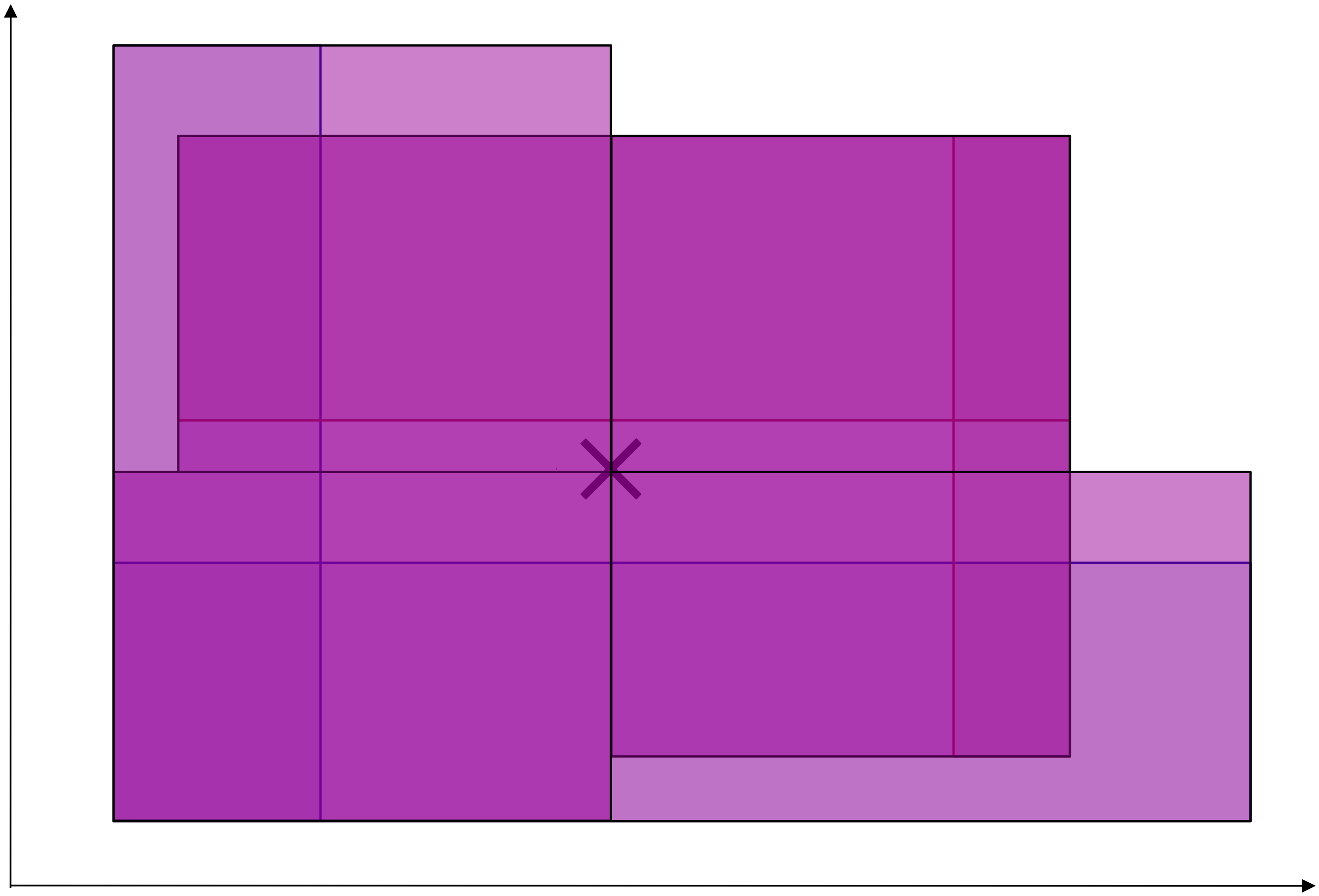
Unification of Two Concepts



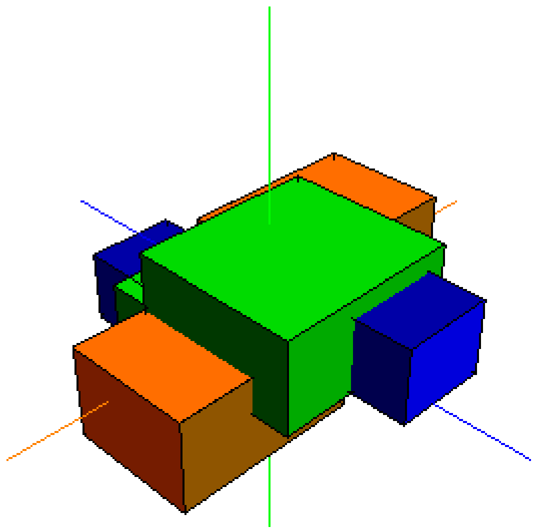
Unification of Two Concepts

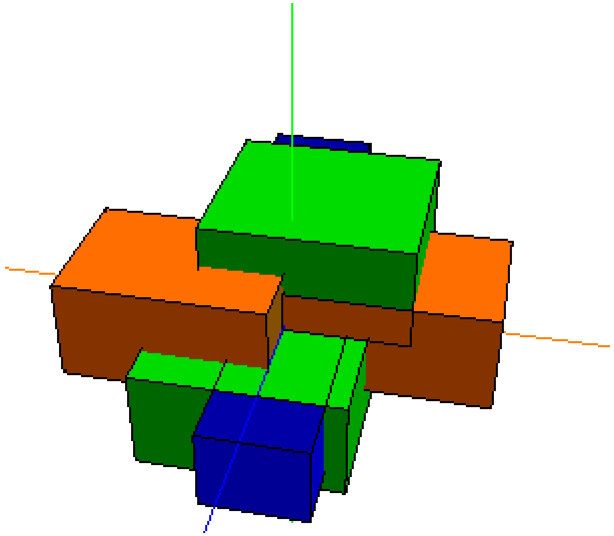


Unification of Two Concepts

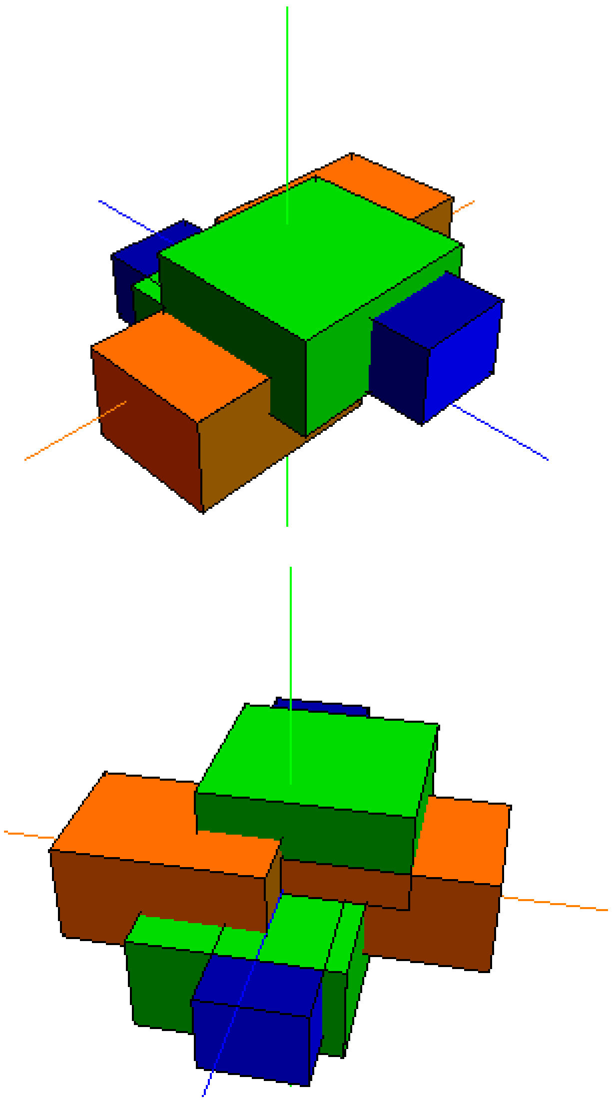


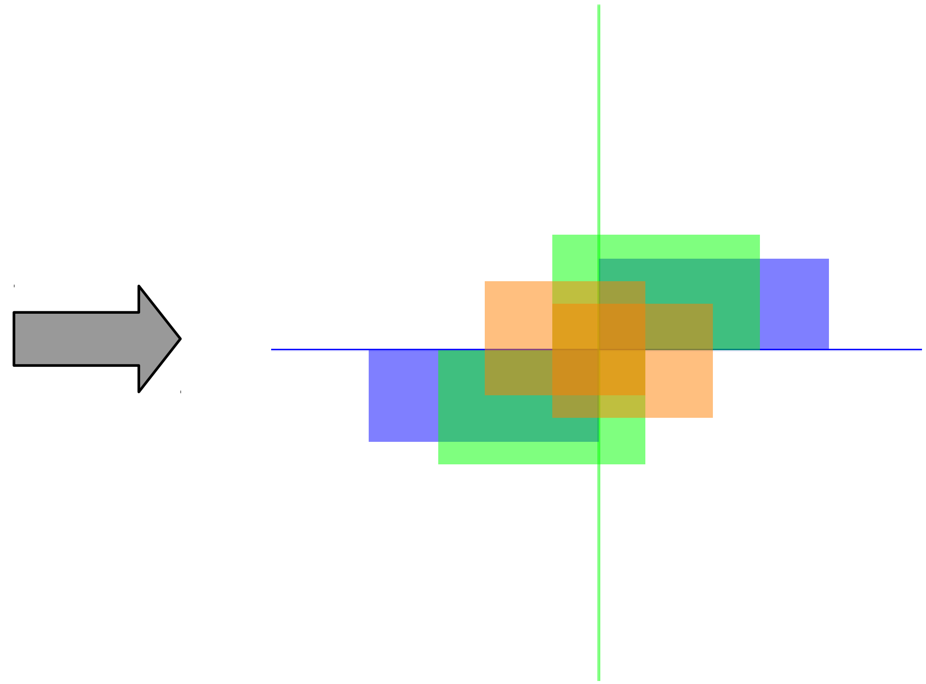
Projection of a Concept





Projection of a Concept





Research Contributions

Research Contributions

- We can encode correlations in a geometric way
  - Most formalizations of conceptual spaces ignore cross-domain correlations
  - [Rickard2006] considers correlations, but not in a geometric way

[Rickard2006] Rickard, J. T. A Concept Geometry for Conceptual Spaces. *Fuzzy Optimization and Decision Making*, Springer Science + Business Media, 2006, 5, 311-329

Research Contributions

- We can encode correlations in a geometric way
  - Most formalizations of conceptual spaces ignore cross-domain correlations
  - [Rickard2006] considers correlations, but not in a geometric way
- Easily implementable and computationally efficient
  - Cuboid can be represented by two support points
  - Single constraint: cuboids of a concept must intersect

[Rickard2006] Rickard, J. T. A Concept Geometry for Conceptual Spaces. *Fuzzy Optimization and Decision Making*, Springer Science + Business Media, 2006, 5, 311-329

UNIVERSITÄT OSNABRÜCK

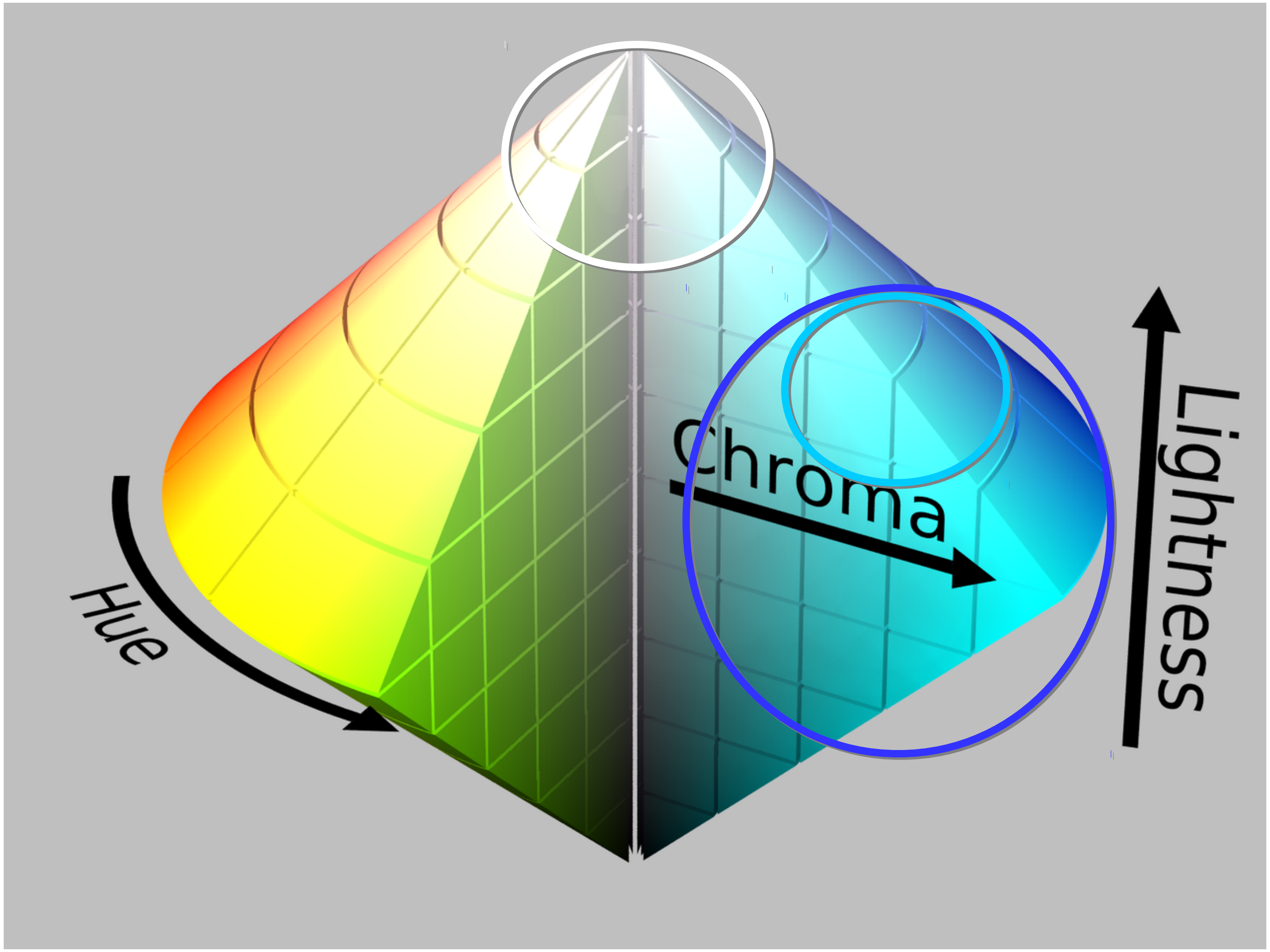
Thank you for your attention!

Questions? Comments? Discussions?

https://www.lucas-bechberger.de

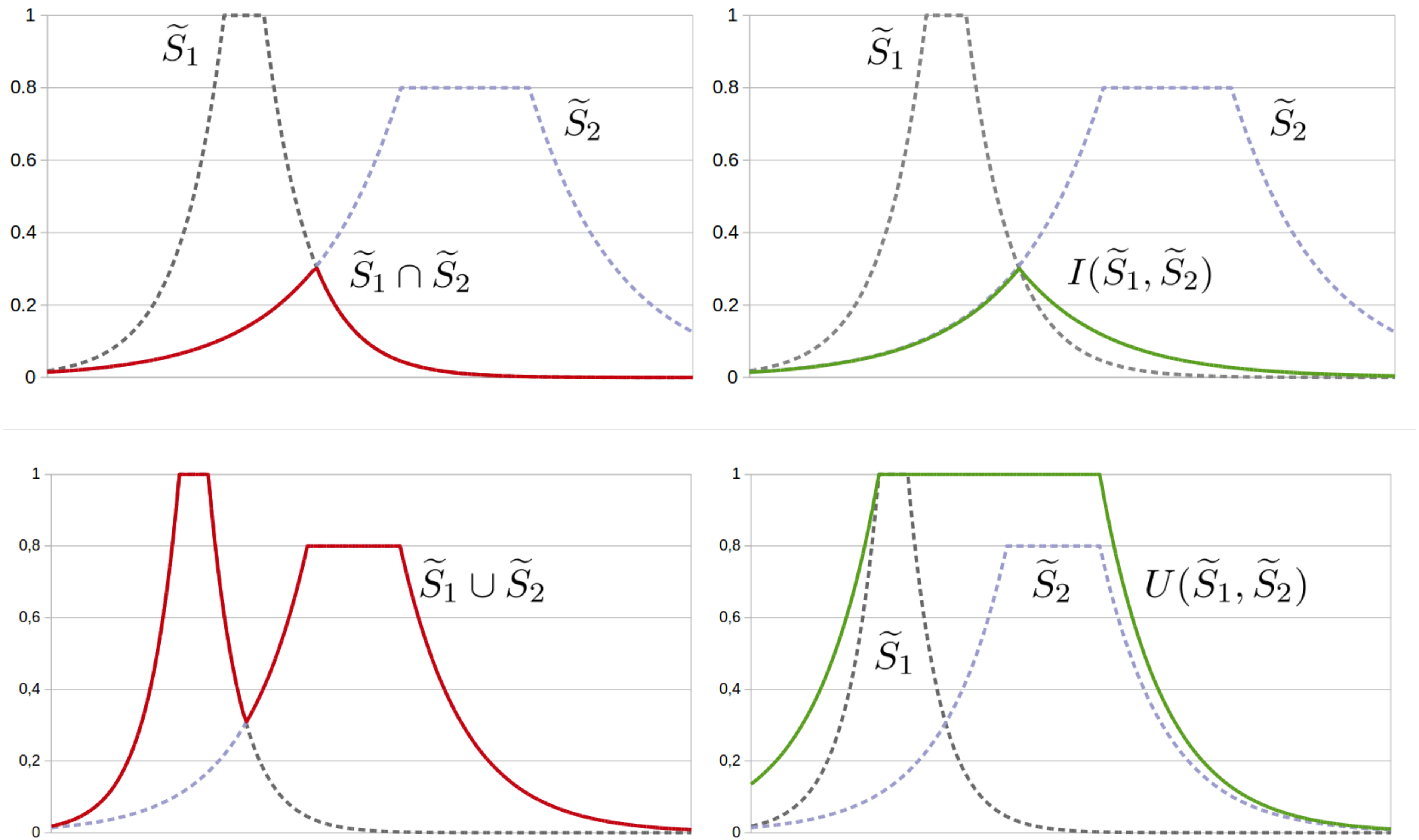
@LucasBechberger

Example: The Color Domain

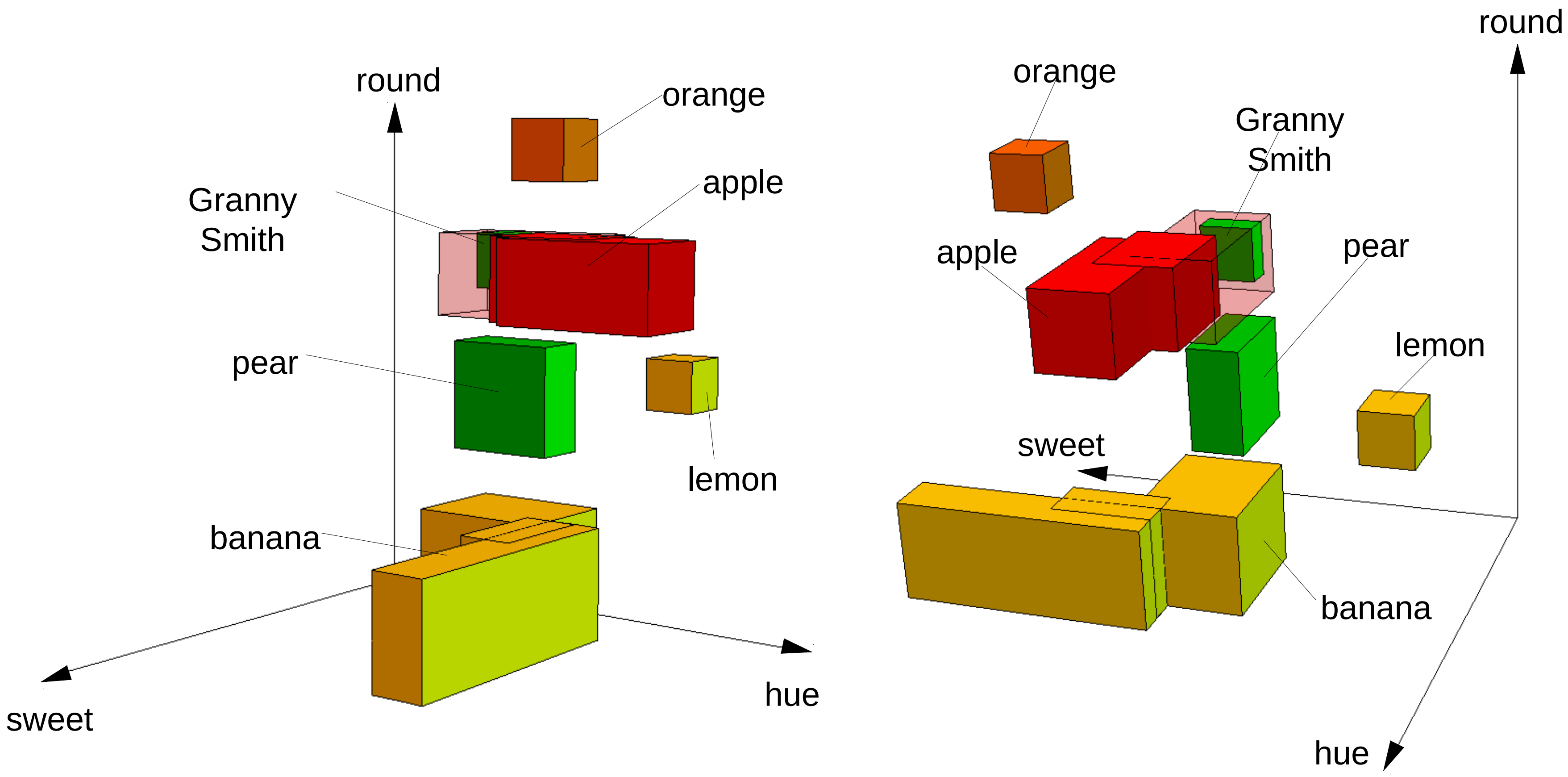


https://en.wikipedia.org/wiki/HSL_and_HSV#/media/File:HSL_color_solid_dblcone_chroma_gray.png

Intersection & Union (Fuzzy Case)



Example: Fruit Space



Envisioned Architecture