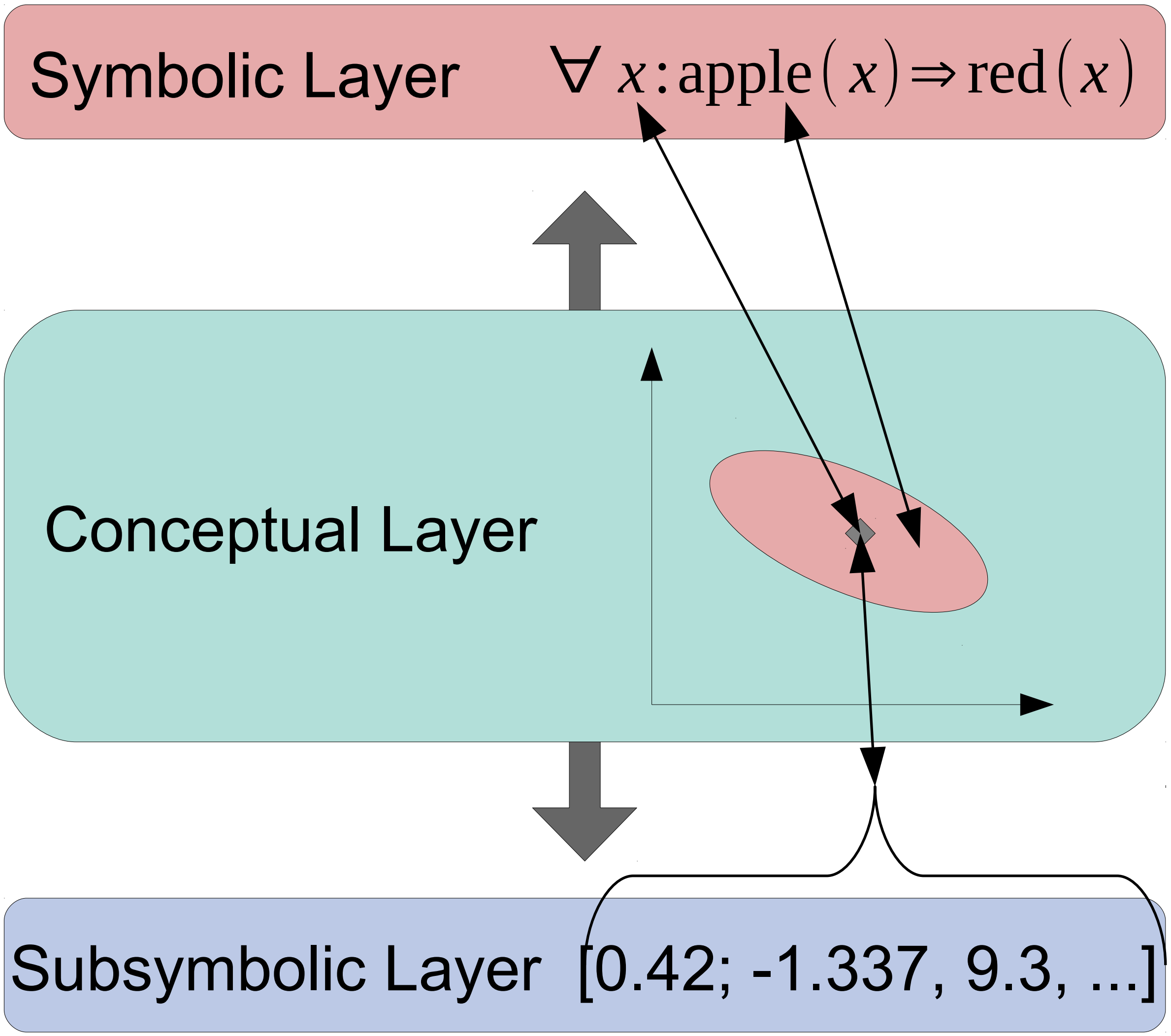
Representing Correlations in Conceptual Spaces

Lucas Bechberger

Institute of Cognitive Science
Osnabrück University

lbechberger@uos.de
https://www.lucas-bechberger.de

Representational Layers

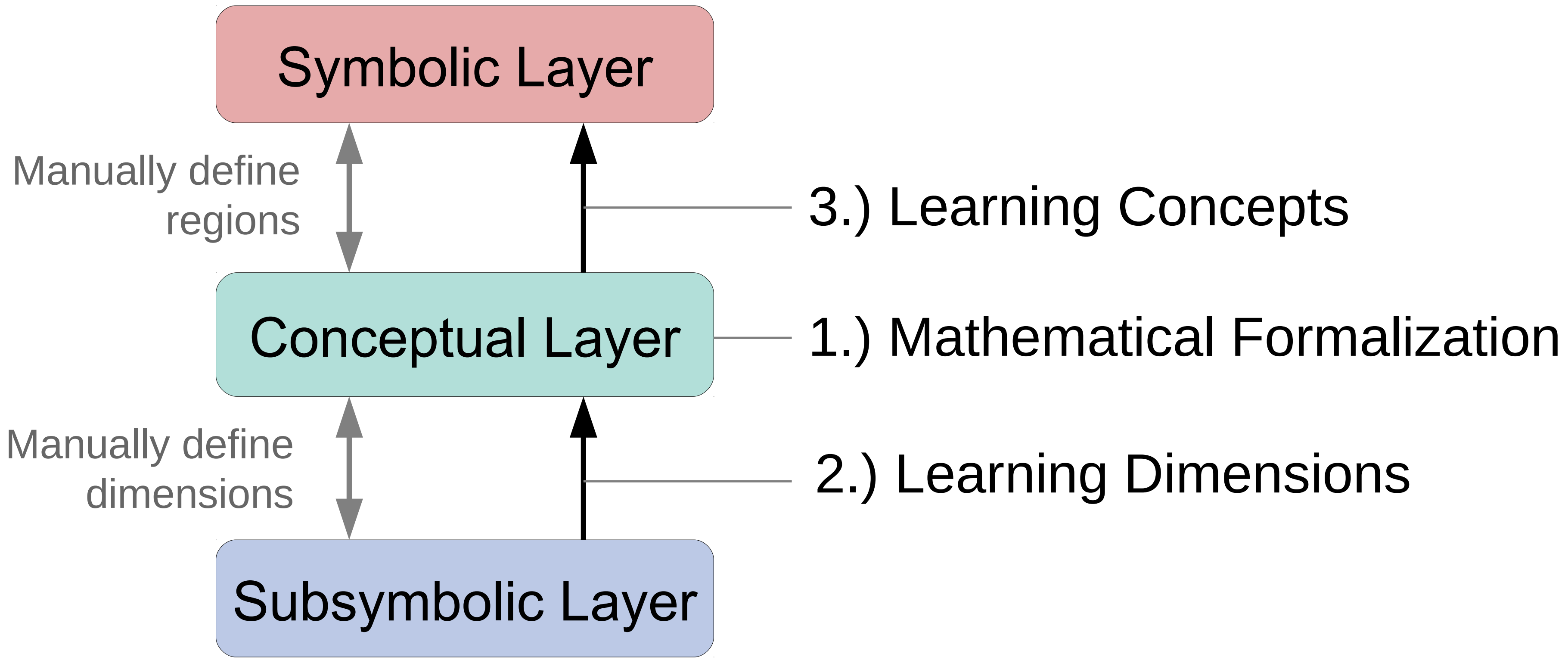


Formal Logics

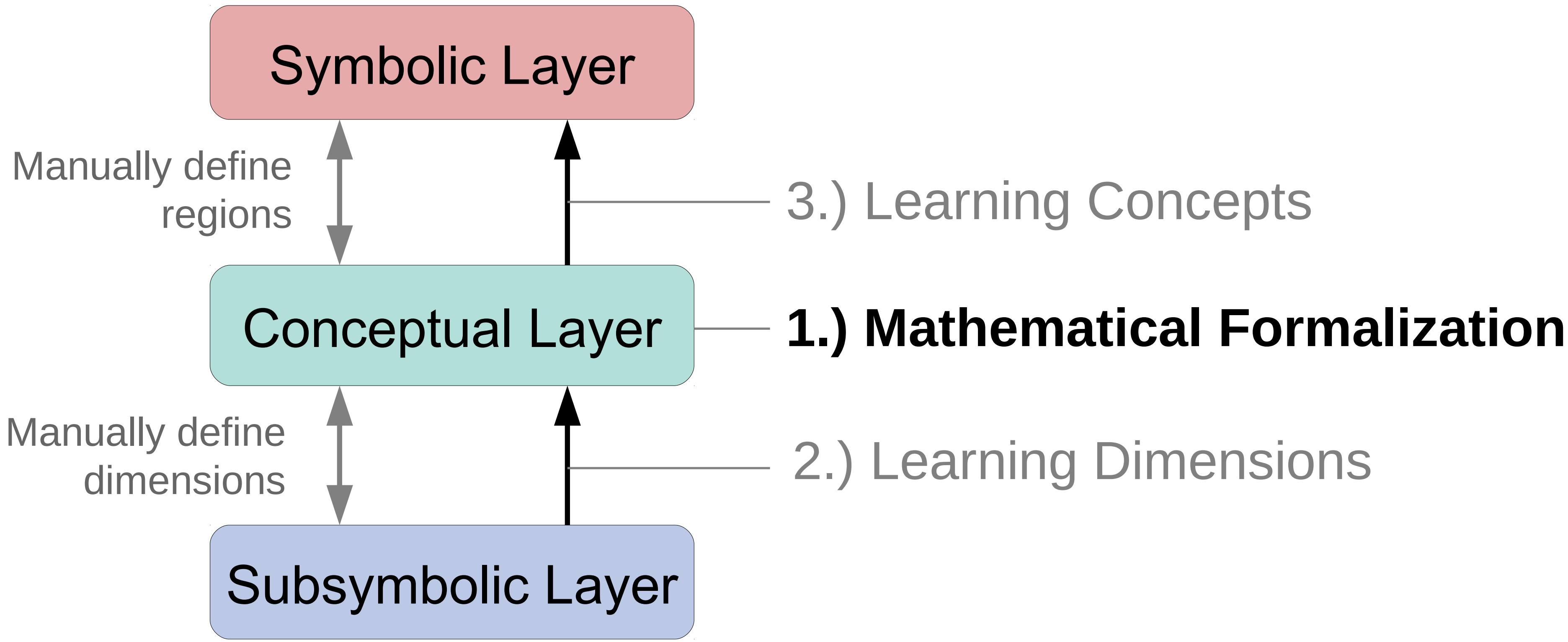
Geometric Representation

Perception, Sensor Values

Conceptual Spaces for AI



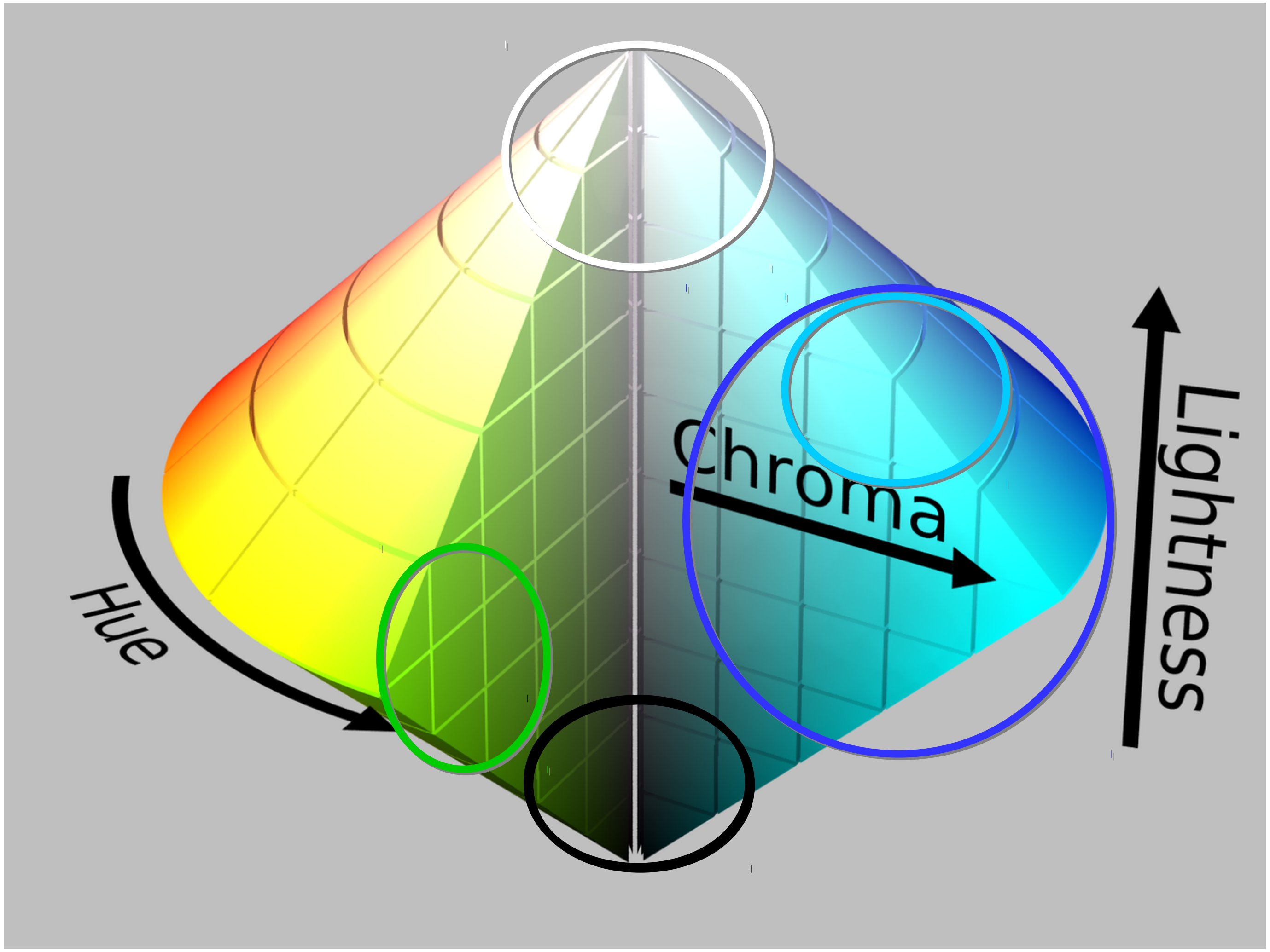
Conceptual Spaces for AI



Conceptual Spaces [Gärdenfors2000]

- Quality dimensions
  - Interpretable ways of judging the similarity of two instances
  - E.g., temperature, weight, brightness, pitch
- Domain
  - Set of dimensions that inherently belong together
  - Color: hue, saturation, and brightness
- Distance in this space is inversely related to similarity
  - Within a domain: Euclidean distance
  - Between domains: Manhattan distance

The Color Domain



https://en.wikipedia.org/wiki/HSL_and_HSV#/media/File:HSL_color_solid_dblcone_chroma_gray.png

Concepts [Gärdenfors2000]

- Property
  - Region within a single domain
  - Examples: “white”, “baby blue”, “hot”, “sour”, “round”

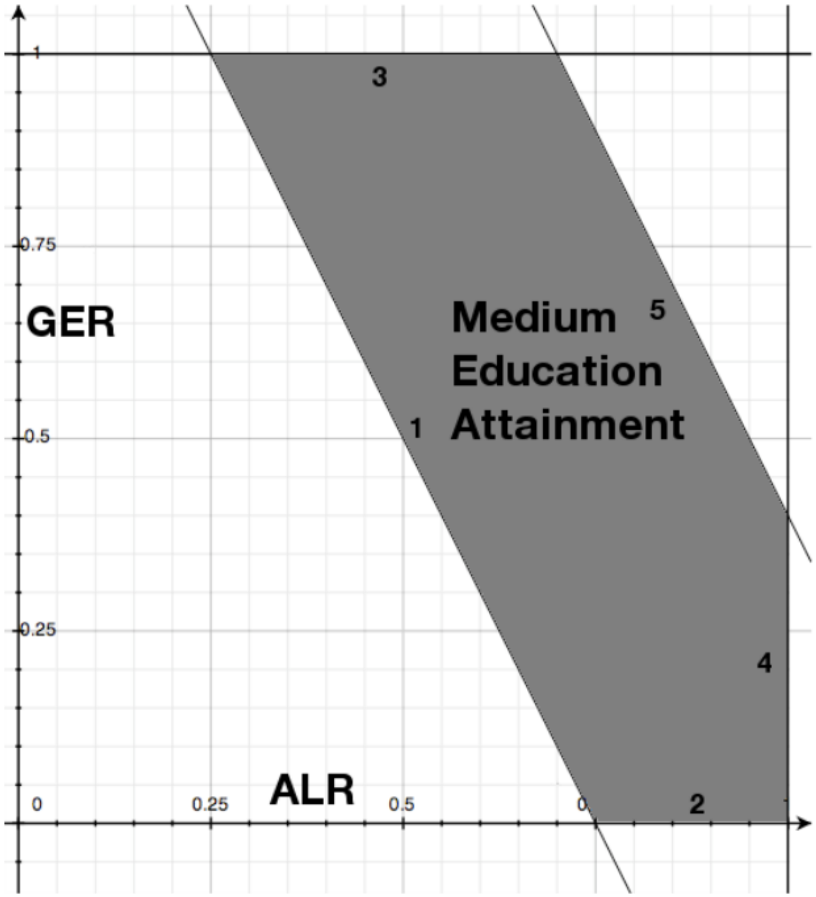
- Concept
  - Spans multiple domains
  - Examples: “apple”, “dog”, “chair”, “university”

- Components of a concept
  - One region per domain
  - Salience weights for the domains
  - **Correlations between the domains**

Criteria for a Good Formalization

- Parametric description of concepts *(Param)*
- Properties and concepts use the same formalism *(Same)*
- Correlations can be encoded *(Corr)*
- Imprecise concept boundaries are possible *(Fuzzy)*
- An implementation is available *(Impl)*

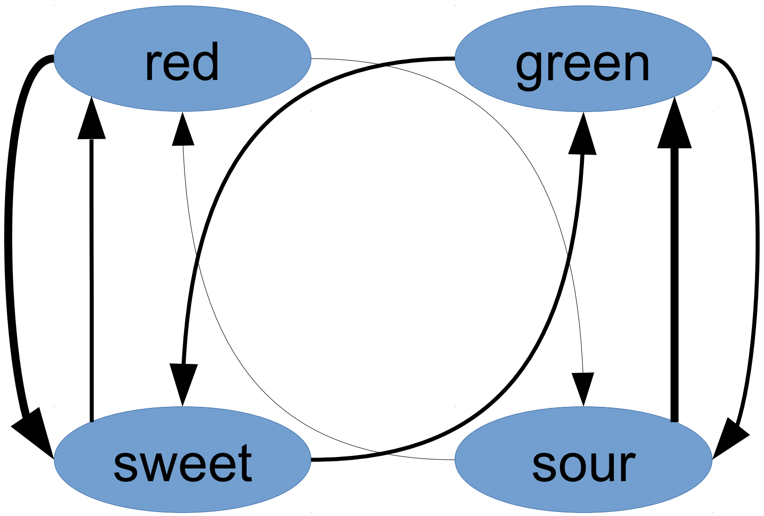
Formalizations [Adams&Raubal2009]



- Property = convex polytope
- Concept = set of properties

Param
Same
Corr
Fuzzy
Impl

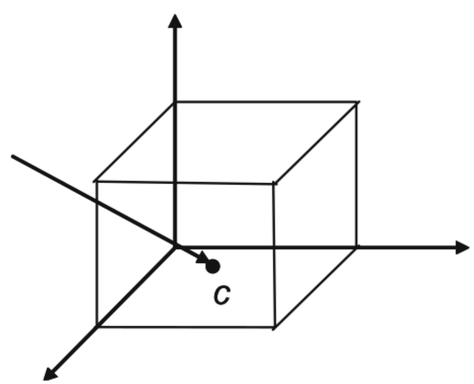
Formalizations [Rickard2006]



	red	green	sweet	sour
red	1.0	0.0	0.9	0.1
green	0.0	1.0	0.4	0.6
sweet	0.7	0.3	1.0	0.0
sour	0.9	0.1	0.0	1.0

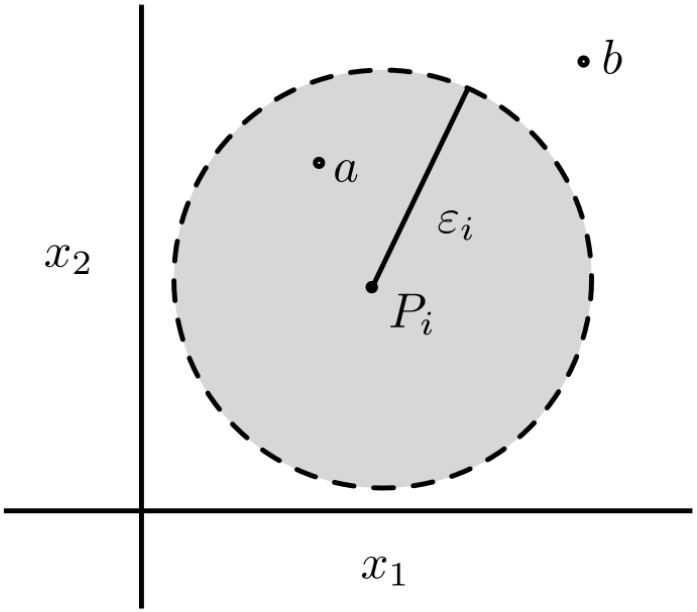
c = (1.0, 0.0, 0.9, 0.1, 0.0, 1.0, 0.4, 0.6, 0.7, 0.3, 1.0, 0.0, 0.9, 0.1, 0.0, 1.0)

$$C = \begin{bmatrix} 1 & C_{12} & \cdots & C_{1N} \\ C_{21} & 1 & \cdots & C_{2N} \\ \vdots & \cdots & \ddots & \vdots \\ C_{N1} & C_{N2} & \cdots & 1 \end{bmatrix}$$

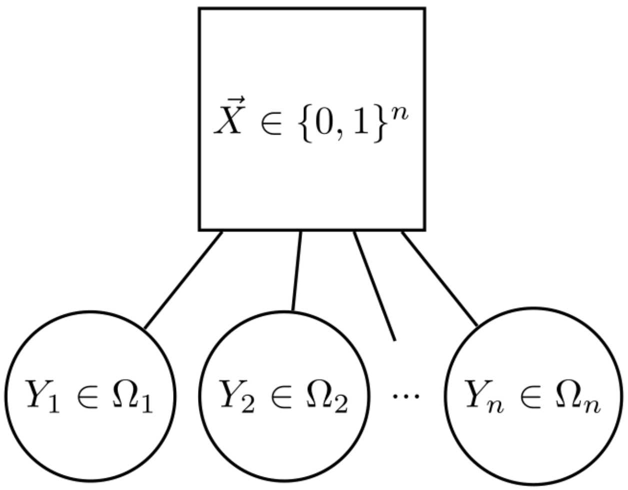


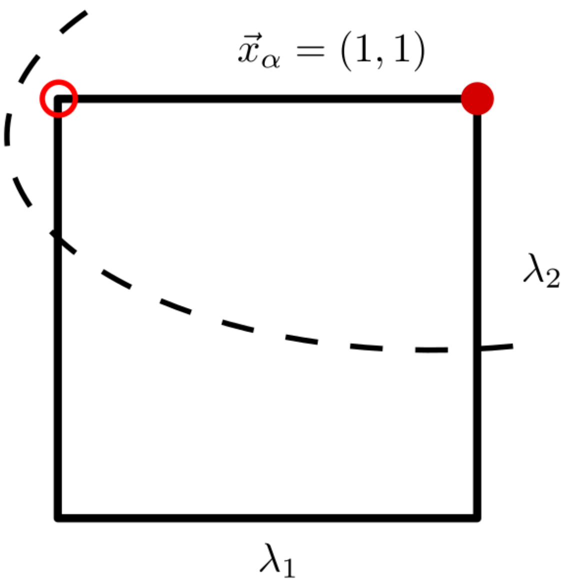
Param
Same
Corr
Fuzzy
Impl

Formalizations [Lewis&Lawry2016]



$$\mu_{L_i}(x) = P(\varepsilon_i : d(x, P_i) \leq \varepsilon_i) = \int_{d(x,P_i)}^{\infty} \delta_i(\varepsilon_i) d\varepsilon_i$$

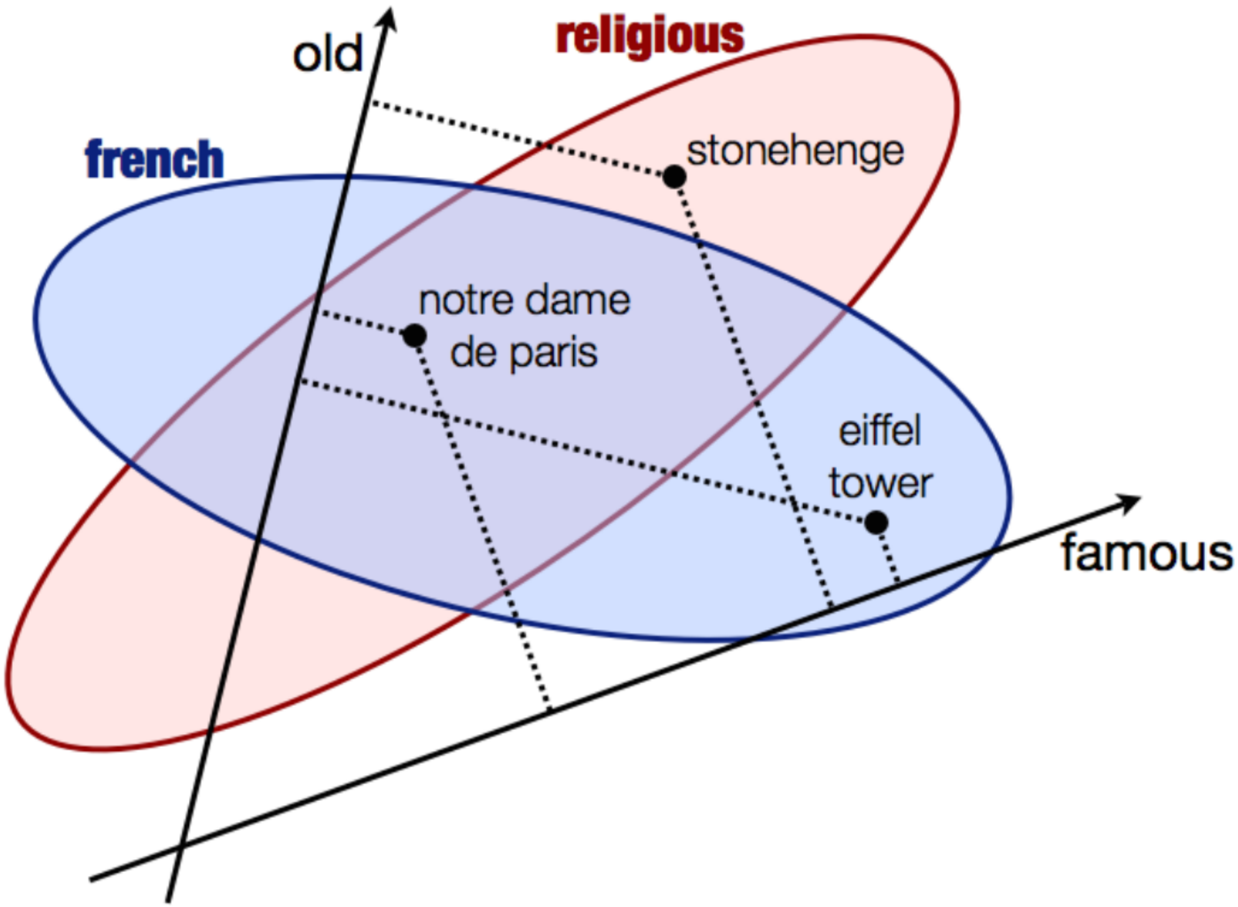




Param
Same
Corr
Fuzzy
Impl

Formalizations [Derrac&Schockaert2015]

- Extract conceptual spaces from textual data
- Find interpretable directions (not necessarily orthogonal)



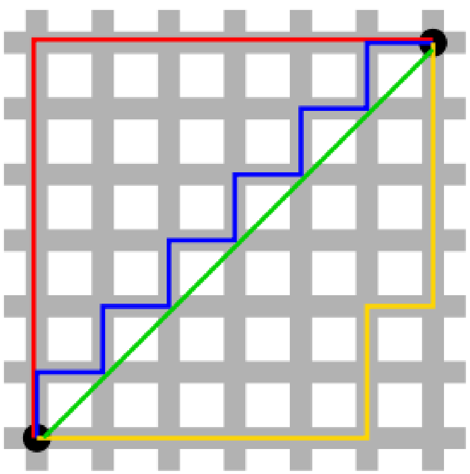
Param
Same
Corr
Fuzzy
Impl

Taking Stock

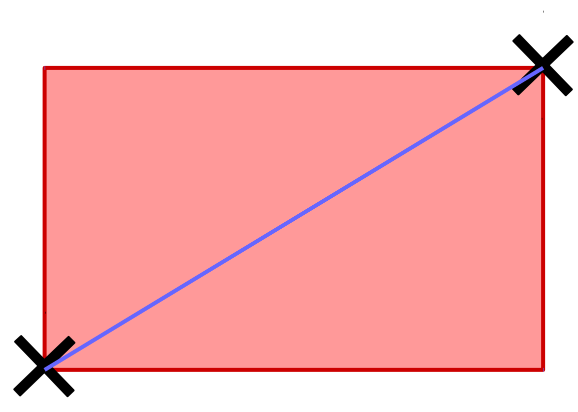
Adams & Raubal	Rickard	Lewis & Lawry	Derrac & Schockaert
Param	Param	Param	Param
Same	Same	Same	Same
Cor	Cor	Cor	Cor
Fuzzy	Fuzzy	Fuzzy	Fuzzy
Impl	Impl	Impl	Impl

Betweenness

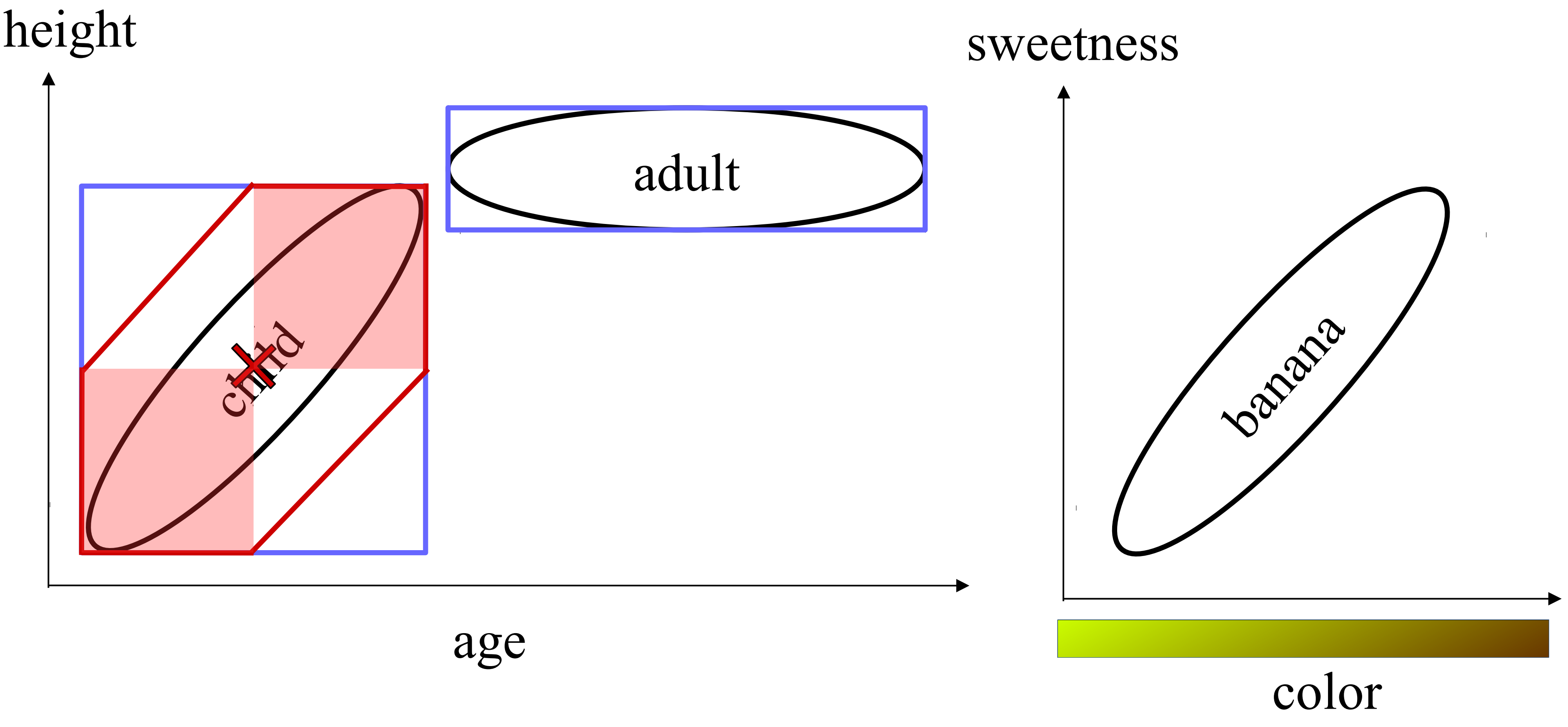
- B(x,y,z) :↔ d(x,y) + d(y,z) = d(x,z)
- Convex region C: $\forall\, x,z \in C : \forall\, y : B(x,y,z) \Rightarrow y \in C$
- Star-shaped region S: $\exists\, p \in S : \forall\, z \in S : \forall\, y : B(p,y,z) \Rightarrow y \in S$



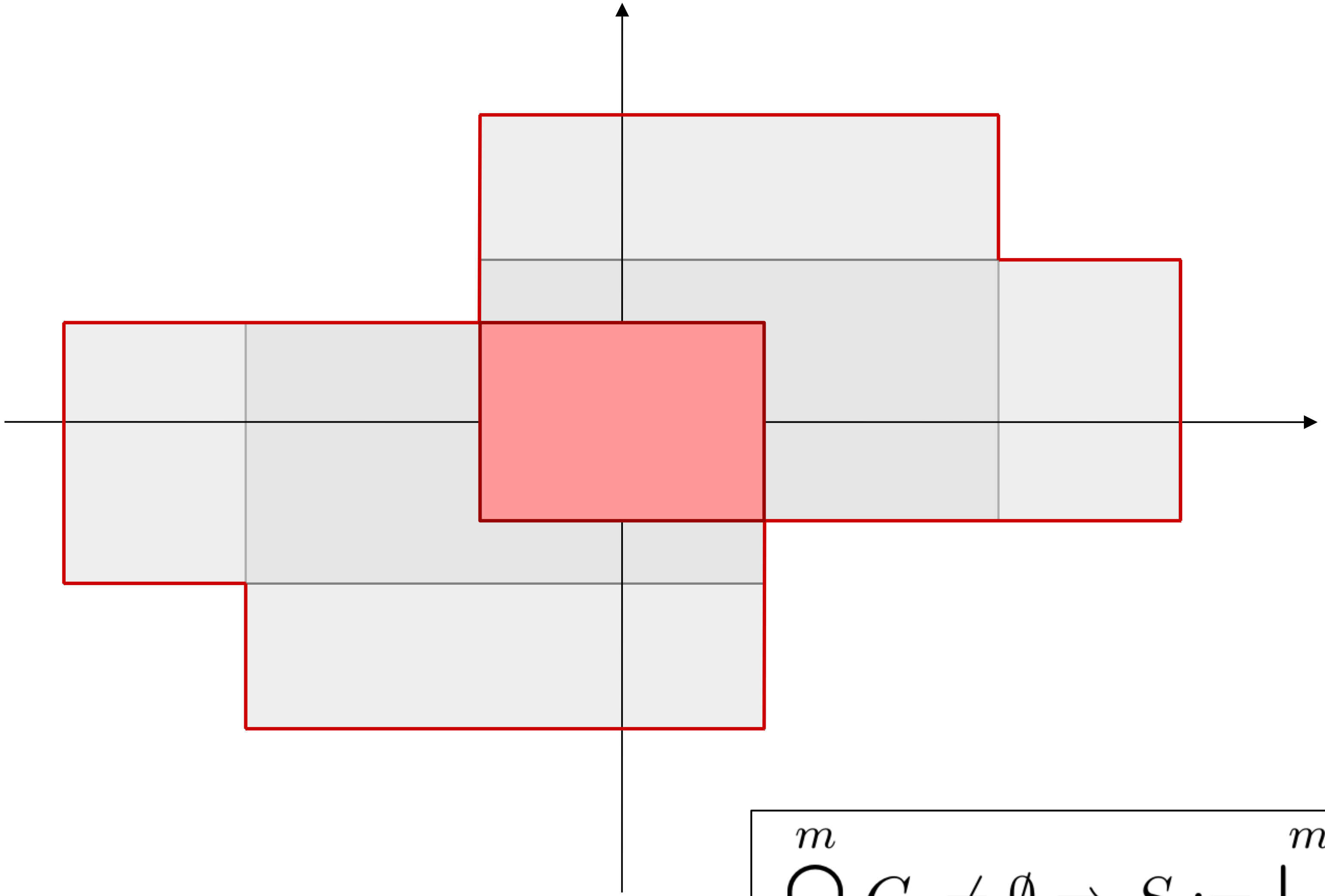
https://en.wikipedia.org/wiki/Taxicab_geometry#/media/File:Manhattan_distance.svg



Convexity and Manhattan distance

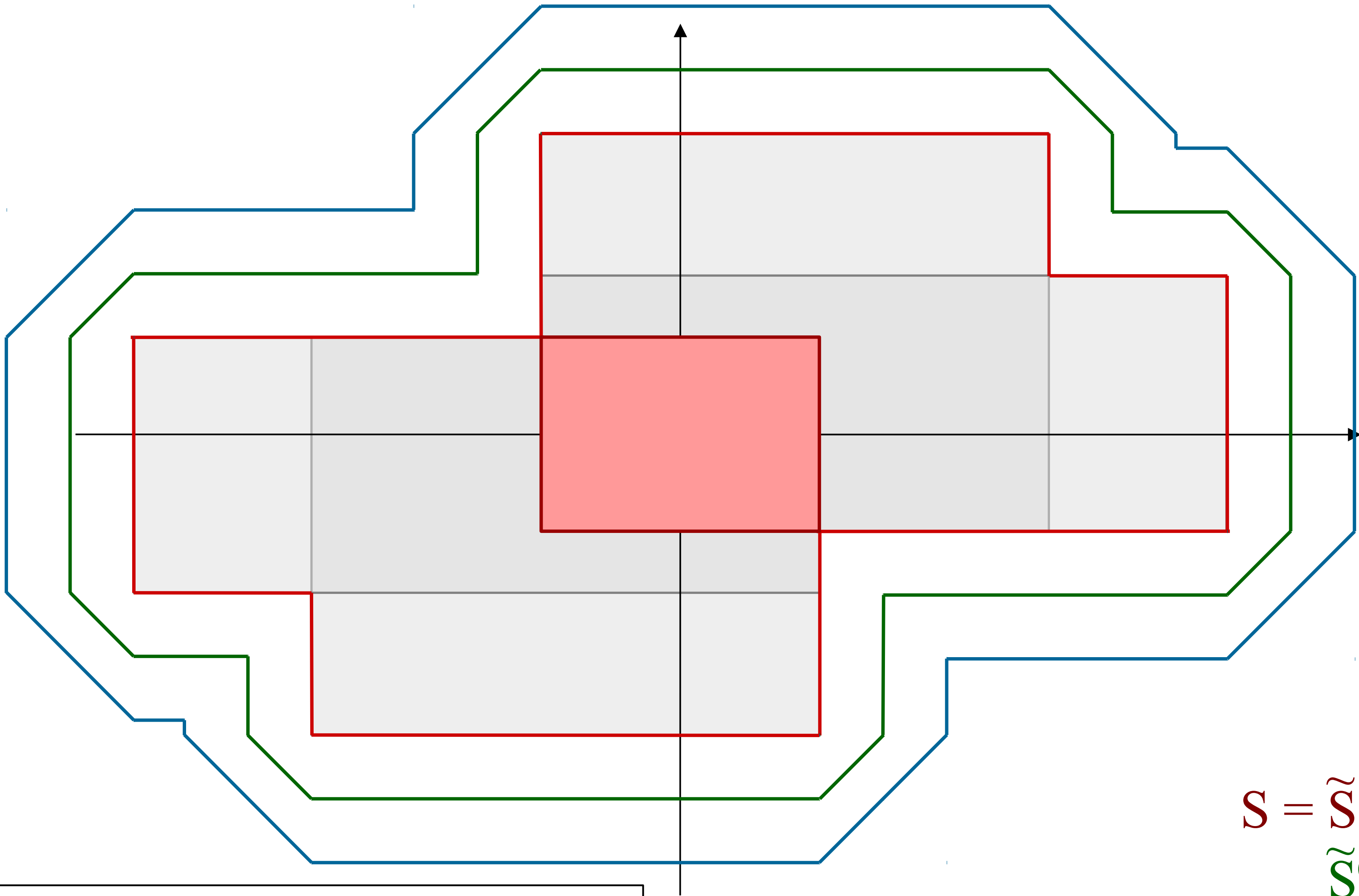


Formalizing Star-Shaped Concepts



$$\bigcap_{i=1}^{m} C_i \neq \emptyset \Rightarrow S := \bigcup_{i=1}^{m} C_i$$

Formalizing Star-Shaped Concepts



$S = \widetilde{S}^{1.0}$

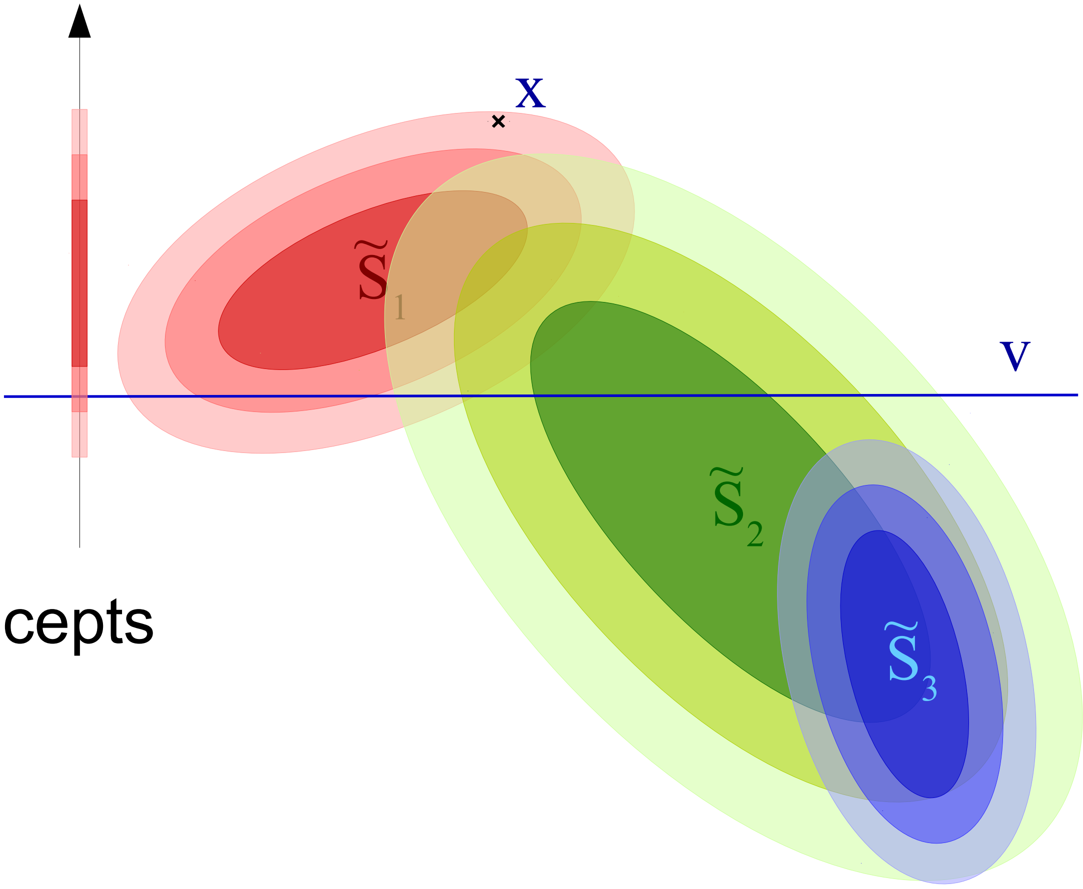
$\widetilde{S}^{0.5}$

$\widetilde{S}^{0.25}$

$$\mu_{\widetilde{S}}(x) = \mu_0 \cdot \max_{y \in S}\left(e^{-c \cdot d_C^{\Delta_S}(x,y,W)}\right)$$

Operations on Concepts

- Basic
  - Membership
- Concept Creation
  - Intersection
  - Unification
  - Projection
  - Cut
- Relations Between Concepts
  - Size
  - Subsethood
  - Implication
  - Similarity
  - Betweenness



Formalization – Summary

- Concepts are represented in parametric way
- We use the same formalism for concepts and properties
- We can encode correlations within a concept in a geometric way
- We have imprecise concept boundaries

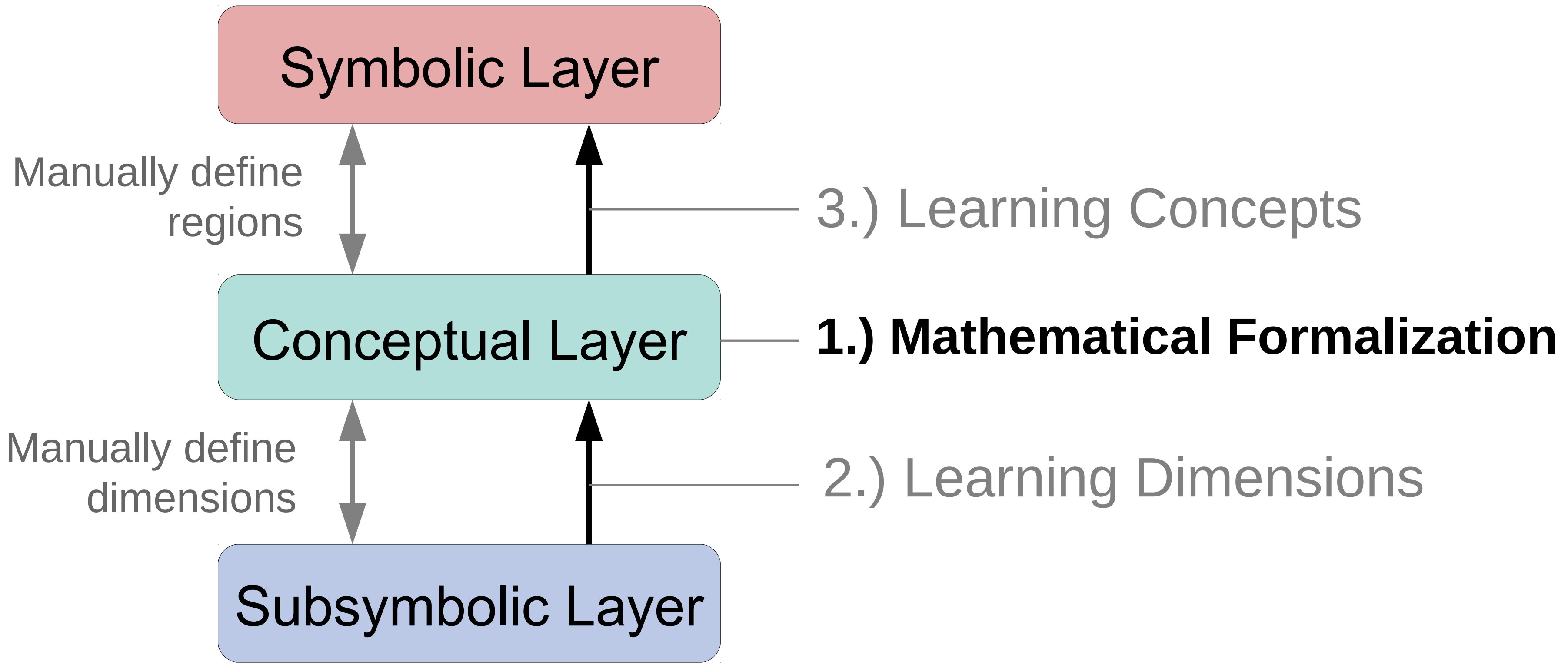
- Quite straightforward to implement
  - Represent each cuboid by two support points
  - Single constraint: cuboids must intersect
  - https://github.com/lbechberger/ConceptualSpaces

- Comprehensive list of supported operations

Param
Same
Corr
Fuzzy
Impl

DEMO TIME!

Conceptual Spaces for AI



Thank you for your attention!

Questions? Comments? Discussions?

https://www.lucas-bechberger.de

@LucasBechberger

References

- [Gärdenfors 2000]
  - Gärdenfors, P. “Conceptual Spaces: The Geometry of Thought”. MIT press, 2000.
- [Rickard2006]
  - Rickard, J. T. “A Concept Geometry for Conceptual Spaces”. Fuzzy Optimization and Decision Making, 2006
- [Adams&Raubal2009]
  - Adams, B. & Raubal, M. “A Metric Conceptual Space Algebra”. 9th International Conference on Spatial Information Theory, Springer Berlin Heidelberg, 2009, 51-68
- [Lewis&Lawry2016]
  - Lewis, M. & Lawry, J. “Hierarchical Conceptual Spaces for Concept Combination”. Artificial Intelligence, Elsevier BV, 2016, 237, 204-227
- [Derrac&Schockaert2015]
  - Derrac, J. & Schockaert, S. “Inducing Semantic Relations from Conceptual Spaces: A Data-Driven Approach to Plausible Reasoning”. Artificial Intelligence, Elsevier BV, 2015, 228, 66-94